Rationalizing the Denominator
Extension Lesson 10.1

In Section 6.1, you used properties to simplify radical expressions. A radical expression is in simplest form when the following are true:

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The process of removing a radical from the denominator of a fraction is called rationalizing the denominator. This can be done two ways.

1. Multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator.
2. Multiply the fraction by the conjugate of the denominator.

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$$= \frac{\sqrt{3}}{3}$$ Evaluate the square root.

Using Conjugates

The binomials $a\sqrt{b}+c\sqrt{d}$ *and* $a\sqrt{b}-c\sqrt{d}$ are called conjugates. Notice how the sign in the middle is different. Multiplying a radical expression by its conjugate will remove the radical sign.

$$(a\sqrt{b}+c\sqrt{d})(\,a\sqrt{b}-c\sqrt{d})$$

$$=(a\sqrt{b})(a\sqrt{b})+(a\sqrt{b})(-c\sqrt{d})+(c\sqrt{d})(a\sqrt{b})+(c\sqrt{d})(-c\sqrt{d})$$

$$=a^2\sqrt{b^2}-ac\sqrt{bd}+ac\sqrt{bd}-c^2\sqrt{d^2}$$

$$=a^2\sqrt{b^2}-c^2\sqrt{d^2}$$

$$=a^2b-c^2d$$

Example: $$\frac{1}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}}=\frac{3+\sqrt{2}}{3^2-(\sqrt{2})^2}=\frac{3+\sqrt{2}}{9-2}=\frac{3+\sqrt{2}}{7}$$