SOLVING QUADRATIC FUNCTIONS USING THE QUADRATIC FORMULA

Lesson 9.4

Another way to solve quadratic equations is to use the *quadratic formula*.

Key Idea

Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a \neq 0$ and $b^2 - 4ac \geq 0$. This is called the **quadratic formula**.

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So, the solutions are $x = \dfrac{5+1}{4} = \dfrac{3}{2}$ and $x = \dfrac{5-1}{4} = 1$.

The expression $b^2 - 4ac$ in the quadratic formula is the **discriminant**.

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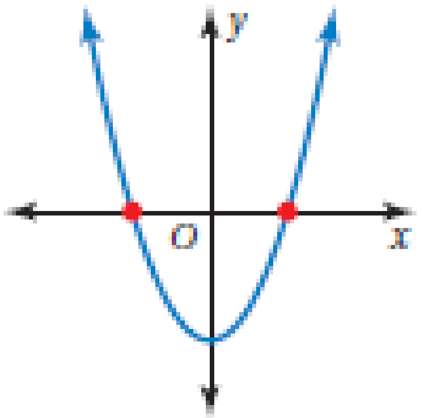
$b^2 - 4ac > 0$



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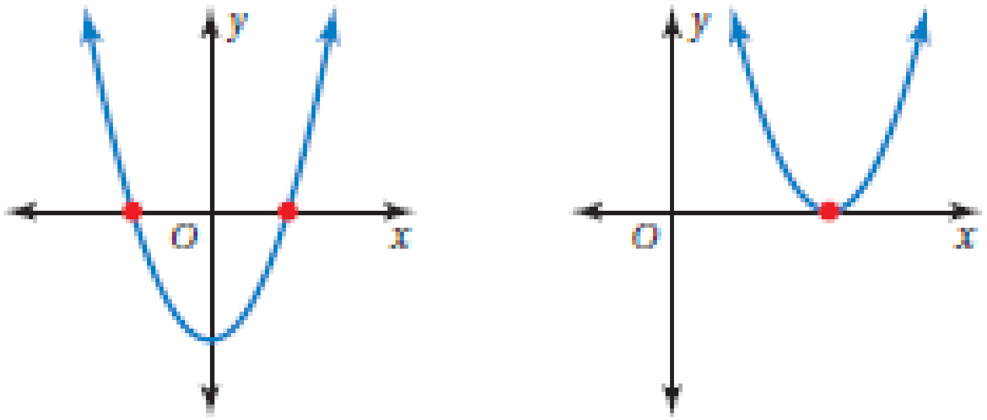

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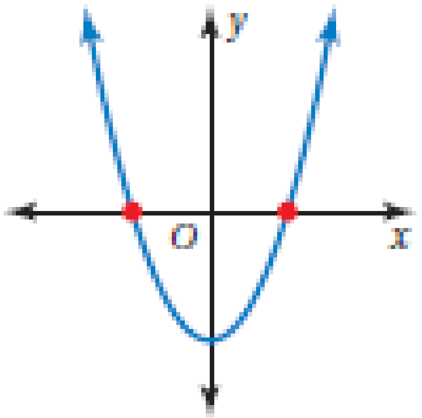

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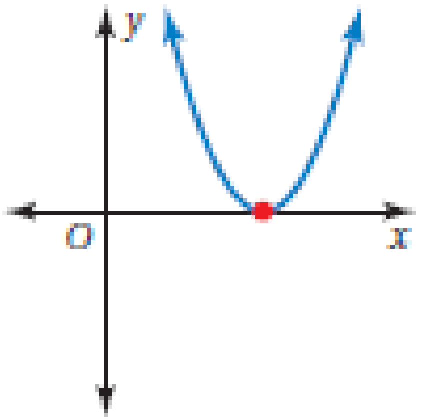

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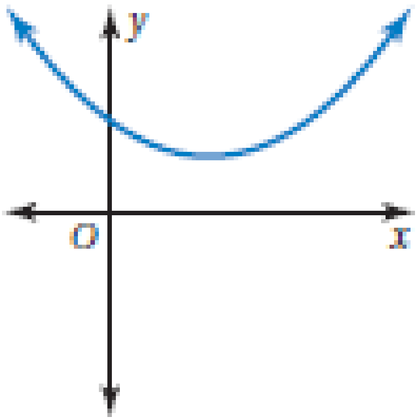

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The discriminant is less than 0, so the equation has no real solutions.