SOLVING QUADRATIC FUNCTIONS BY COMPLETING THE SQUARE

Lesson 9.3

Another method for solving quadratic equations is **completing the square**. In this method, a constant c is added to the expression $x^2 + bx$ so that $x^2 + bx + c$ is a perfect square trinomial.

Key Idea

Words To complete the square for an expression of the form $x^2 + bx$, follow these steps.

Step 1: Find $\frac{b}{2}$.

Step 2: Square the result in Step 1, $\left(\frac{b}{2}\right)^2$.

Step 3: Add the result in Step 2 to $x^2 + bx$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2$$

Factor the resulting expression as the square of a binomial.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

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Complete the square for each expression. Then factor the trinomial.

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$x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

On Your Own

Complete the square for each expression. Then factor the trinomial.

1. $x^2 + 10x$

$x^2 + 10x + 25 = (x + 5)^2$

2. $x^2 - 4x$

$x^2 - 4x + 4 = (x - 2)^2$

3. $x^2 + 7x$

$x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$

To solve a quadratic equation by completing the square, write the equation in the form $x^2 + bx = d$.

Step 1: Add/subtract the constant to the right side of the equal sign.

Step 2: Take $\frac{b}{2}$, then square it.

Step 3: Add the result in Step 2 to both sides.

Step 4: Factor

Step 5: Take the Square root.

Completing the Square

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Example

$$x^2 - 6x + 5 = 0$$

Step 1: $x^2 - 6x + \square = -5 + \square$

Step 2: $(-3)^2 = 9$

Step 3: $x^2 - 6x + \boxed{9} = -5 + \boxed{9}$

Step 4: $(x-3)^2 = 4$

Step 5: $\sqrt{(x-3)^2} = \pm\sqrt{4}$

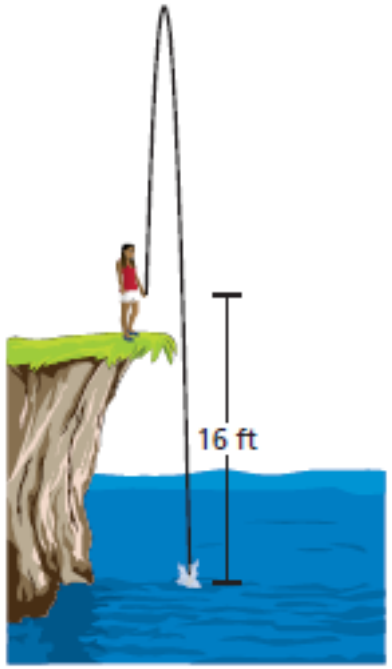
Solve: $x - 3 = \pm 2$

$x = 3+2 \quad \text{or} \quad x = 3-2$

$x = 5 \quad \text{or} \quad x = 1$

EXAMPLE 3 Real-Life Application

You throw a stone from a height of 16 feet with an upward velocity of 32 feet per second. The function $h = -16t^2 + 32t + 16$ gives the height h of the stone after t seconds. When does the stone land in the water?



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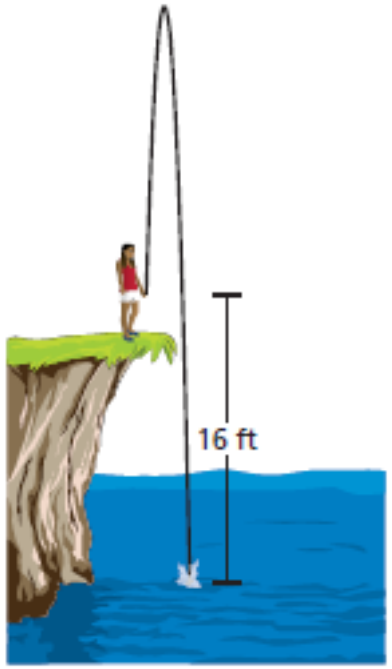

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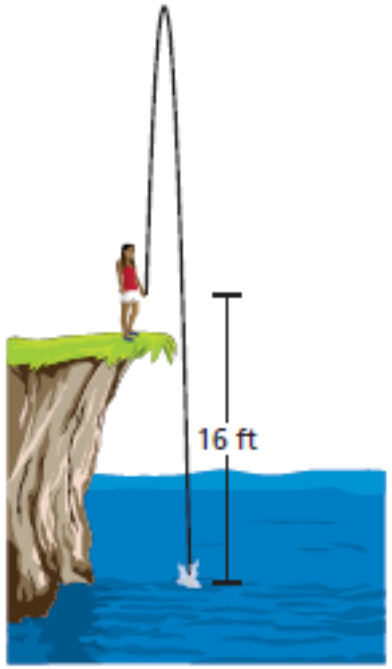

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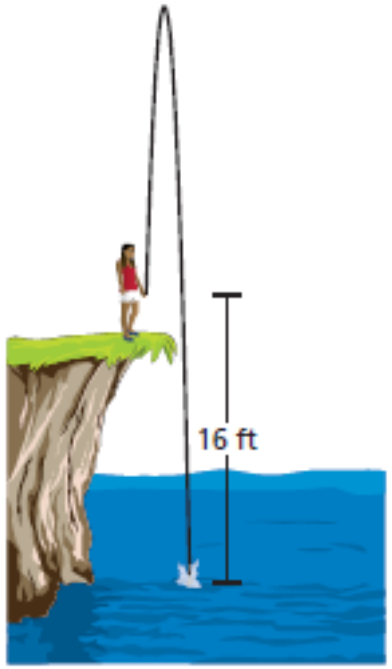

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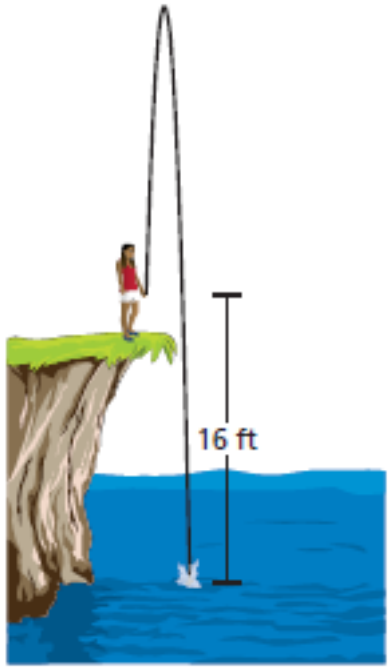

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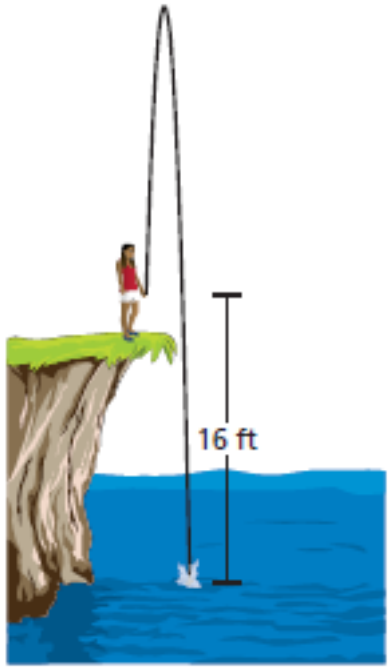

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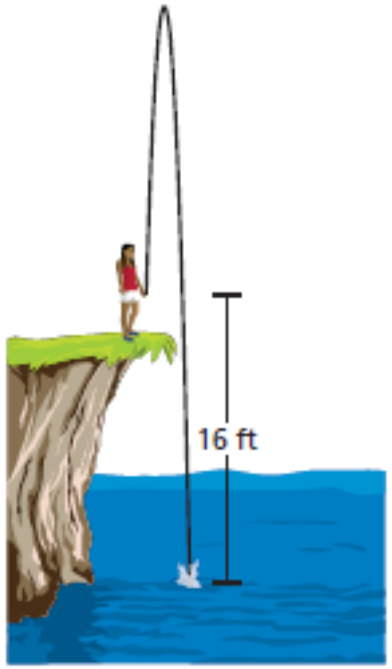

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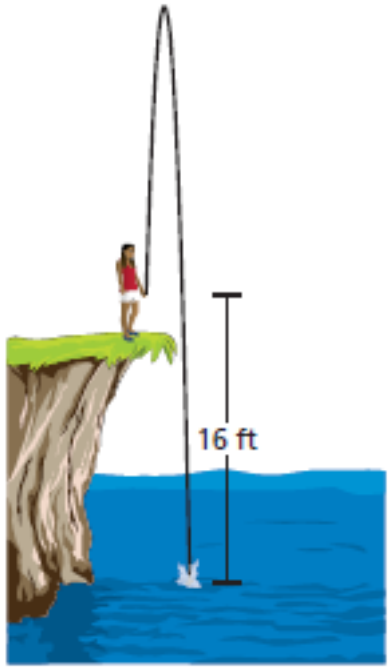

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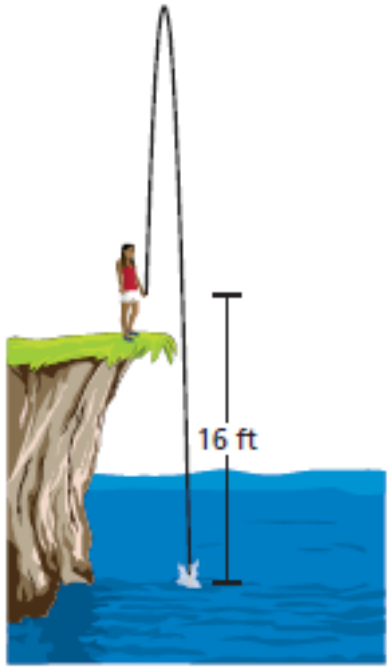

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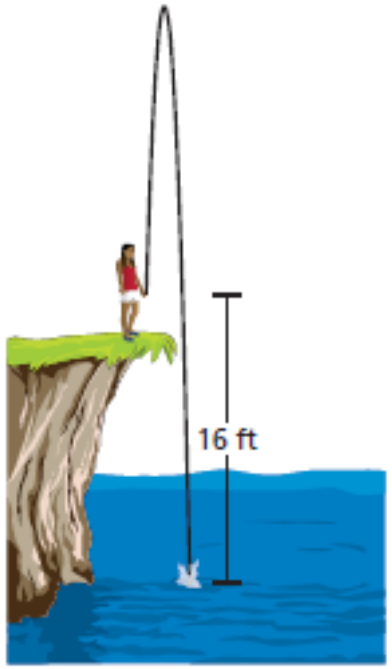

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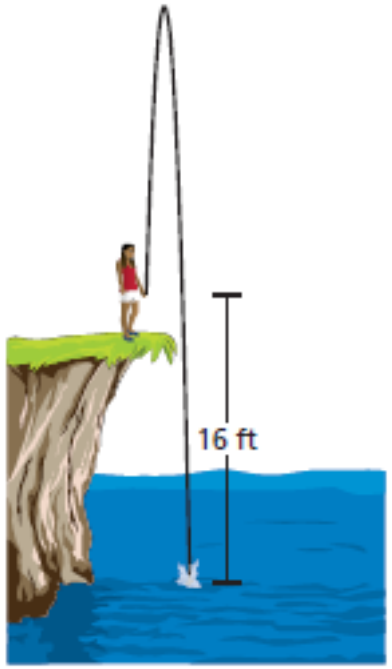

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The stone lands in the water in about 2.4 seconds.

Homework

Textbook pages 472 & 473:

6-17, 26, 33-36