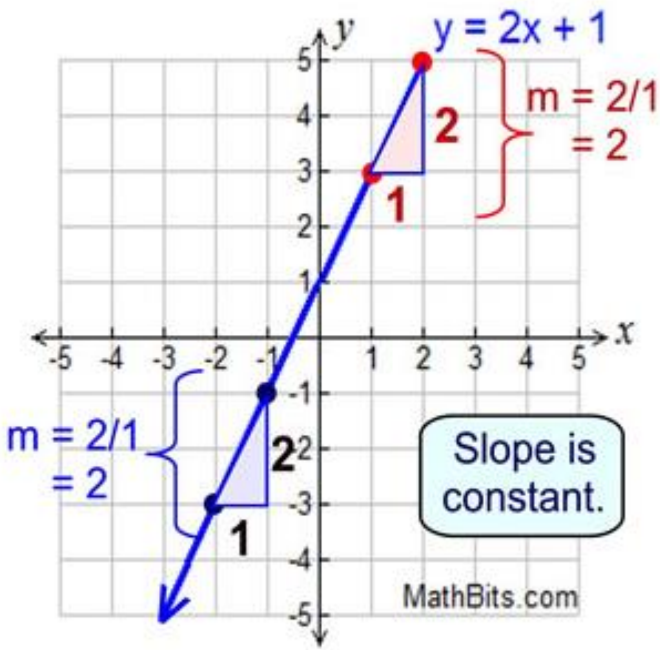
Comparing Graphs of Functions
Lesson 8.5 Extension

Linear Functions:

You are already familiar with the concept of "average rate of change".
When working with **straight lines** (*linear functions*) you saw the "average rate of change" to be:

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = m = SLOPE$$

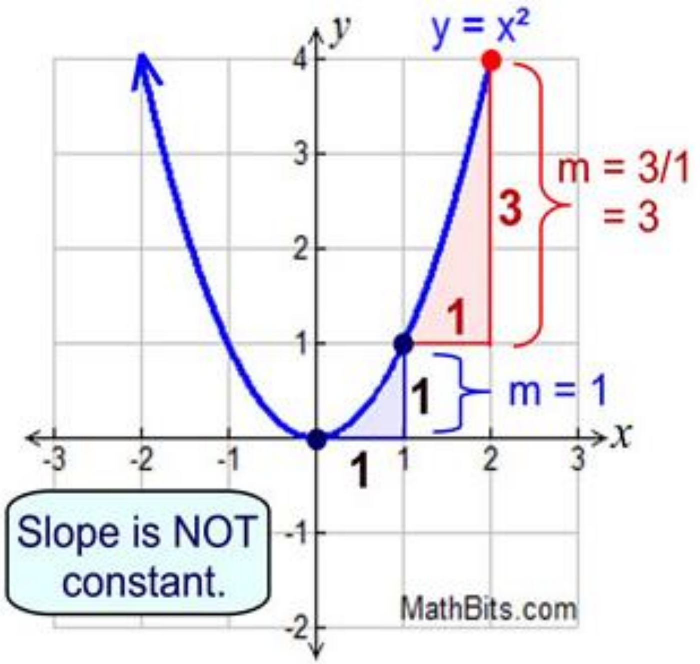


A special circumstance exists when working with straight lines (*linear functions*), in that the "average rate of change" (the slope) is constant. No matter where you check the slope on a straight line, you will get the same answer.

Non-Linear Functions:

When working with *non-linear functions,* the "average rate of change" is not constant.

The process of computing the "average rate of change", however, remains the same as was used with straight lines: two points are chosen, and $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\text{rise}}{\text{run}}$ is computed.



When you find the "average rate of change" you are finding the rate at which (how fast) the function's *y*-values (output) are changing as compared to the function's *x*-values (input).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \textit{becomes} \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example 1: Finding average rate of change from a table.

Function $f(x)$ is shown in the table at the right.
Find the average rate of change over the interval $1 \leq x \leq 3$.

x	$f(x)$
0	1
1	4
2	9
3	16

If the interval is $1 \leq x \leq 3$,then you are examining the points (1,4) and (3,16).

Substitute into the formula: $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

$$= \frac{16-4}{3-1}$$

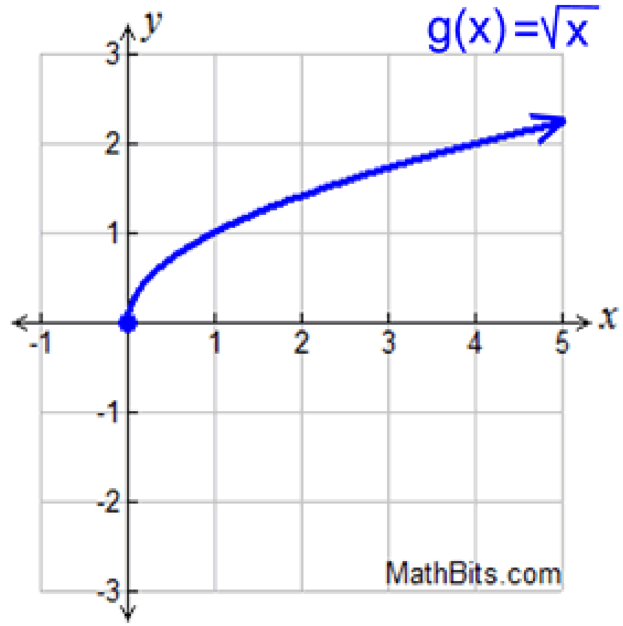
$$= \frac{12}{2}$$

$$= \frac{6}{1}$$

The average rate of change is 6 over 1, or just 6.
The y-values change 6 units every time the x-values change 1 unit, on this interval.

Example 2: Finding average rate of change from a graph.

Function $g(x)$ is shown in the graph at the right. Find the average rate of change over the interval $1 \leq x \leq 4$.



If the interval is $1 \leq x \leq 4$, then you are examining the points (1,1) and (4,2).

Substitute into the formula: $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$= \frac{2-1}{4-1}$$

$$= \frac{1}{3}$$

The average rate of change is 1 over 3, or just 1/3.
The y-values change 1 unit every time the x-values change 3 units, on this interval.

Example 3

Finding the average rate of change for the function below between $x = 1 \ and \ x = 2$.

$g(x) = (x - 3)^2 - 2$

$g(1) = (1 - 3)^2 - 2$

$g(1) = (-2)^2 - 2$

$g(1) = 4 - 2$

$g(1) = 2$

$(1,2)$

$g(2) = (2 - 3)^2 - 2$

$g(2) = (-1)^2 - 2$

$g(2) = 1 - 2$

$g(2) = -1$

$(2, -1)$

Substitute into the formula: $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$= \frac{-1 - 2}{2 - 1}$$

$$= \frac{-3}{1}$$

The average rate of change between $x = 1$ and $x = 2$ is -3.

Finding the average rate of change from a table.

Function $f(x)$ is shown in the table at the right. Find the average rate of change over the interval $0.5 \leq x \leq 2$.

x Time (seconds)	$f(x)$ Ball height (feet)
0	0
0.5	12
1	16
1.5	12
2	0

Substitute into the formula: $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

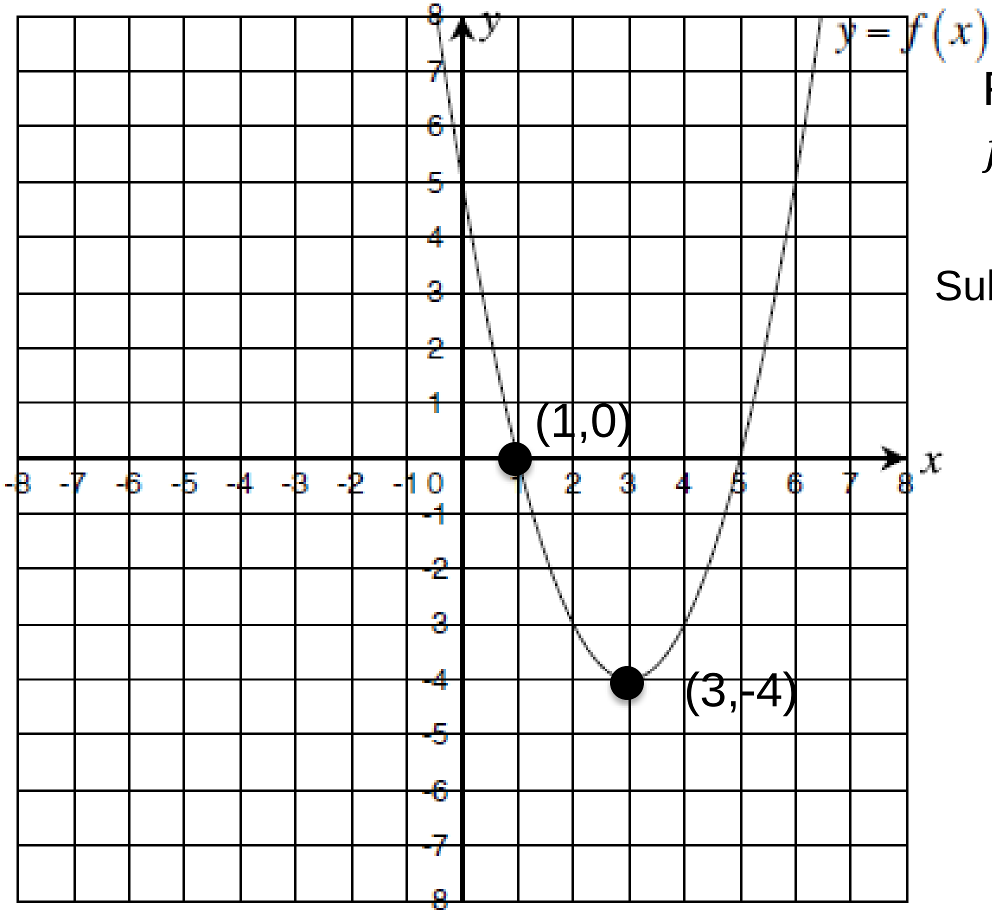
$$= \frac{0-12}{2-0.5}$$

$$= \frac{-12}{1.5}$$

$$= -8$$

The average rate of change is -8 over 1, or just -8.
The y-values change -8 units every time the x-values change 1 unit on this interval.

Consider the quadratic function whose graph is shown.



Find the average rate of change of $f(x)$ from $x = 1$ to $x = 3$.

Substitute into the formula: $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$= \frac{-4 - 0}{3 - 1}$$

$$= \frac{-4}{2}$$

$$= -2$$

The average rate of change is -2, and we can see that the function values for the quadratic function, $y = f(x)$ are decreasing on the interval from $x = 1$ to $x = 3$.

Calculate the average rate of change for the function $f(x) = x^2 + 6x + 9$ between $x = 1$ and $x = 3$.

$f(x) = x^2 + 6x + 9$

$f(3) = (3)^2 + 6(3) + 9$

$f(3) = 36$

$(3,36)$

$f(x) = x^2 + 6x + 9$

$f(1) = (1)^2 + 6(1) + 9$

$f(1) = 16$

$(1,16)$

Substitute into the formula: $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$= \frac{16 - 36}{1 - 3}$$

$$= \frac{-20}{-2}$$

$$= 10$$

The average rate of change of $f(x) = x^2 + 6x + 9$ between $x = 1$ and $x = 3$ is 10.

Homework

Practice 2.2.3

Identifying the Average Rate of Change