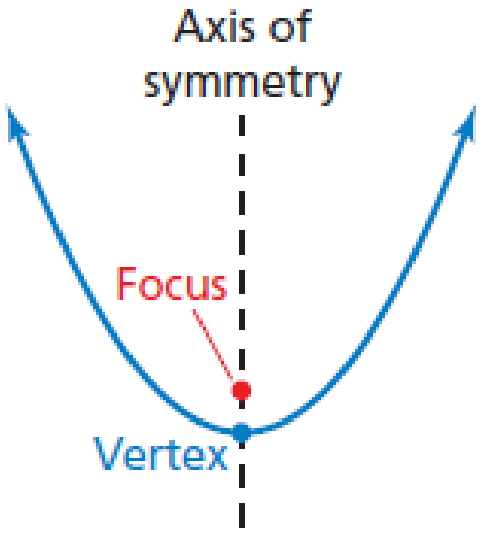
Focus of a Parabola

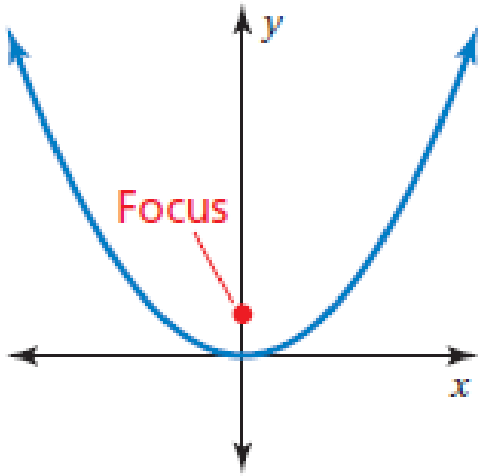
Lesson 8.2

Key Idea

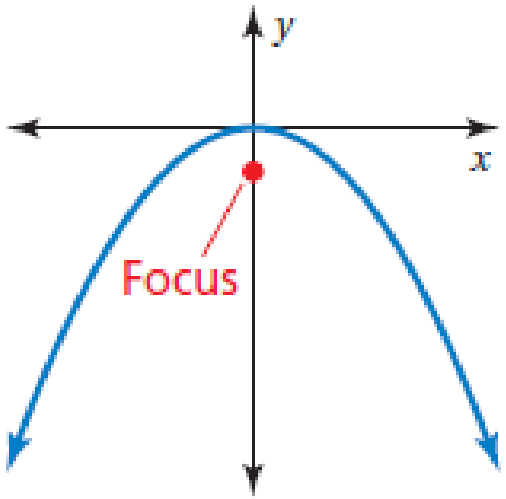
The Focus of a Parabola

The **focus** of a parabola is a fixed point on the interior of a parabola that lies on the axis of symmetry. A parabola "wraps" around the focus.





$y = ax^2, a > 0$



$y = ax^2, a < 0$

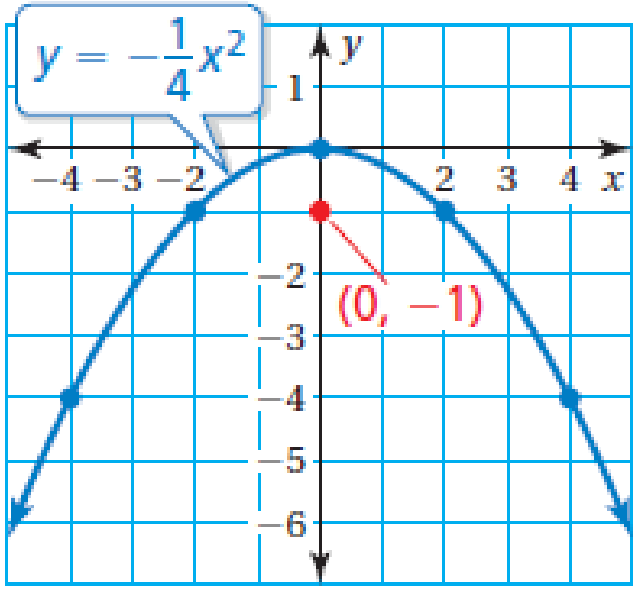
For functions of the form $y = ax^2$, the focus is $\left(0, \frac{1}{4a}\right)$.

EXAMPLE 1 Finding the Focus of a Parabola

Graph $y = -\frac{1}{4}x^2$. Identify the focus.

Step 1: Make a table of values. Then graph.

x	-4	-2	0	2	4
y	-4	-1	0	-1	-4



Step 2: Identify the focus. The function is of the form $y = ax^2$, so $a = -\frac{1}{4}$.

$$\frac{1}{4a} = \frac{1}{4\left(-\frac{1}{4}\right)}$$

$$= \frac{1}{-1}, \text{ or } -1$$

∴ So, the focus of the function $y = -\frac{1}{4}x^2$ is $(0, -1)$.

EXAMPLE 2 Writing an Equation of a Parabola

Write an equation of the parabola with focus (0, 4) and vertex at the origin.

For $y = ax^2$, the focus is $\left(0, \frac{1}{4a}\right)$. Use the given focus, $(0, 4)$, to write an equation to find a.

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$$\frac{1}{4a} = 4$$ Equate the y-coordinates.

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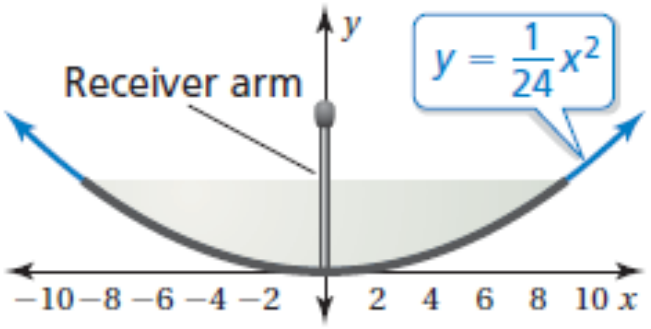
∴ An equation of the parabola is $y = \frac{1}{16}x^2$.

EXAMPLE 3 Real-Life Application

A birdwatcher uses a parabolic microphone to collect and record bird sounds. The cross section of the microphone can be modeled by $y = \frac{1}{24}x^2$, where x and y are measured in inches. The focus is located at the end of the receiver arm. What is the length of the receiver arm?



The arm length is the distance from the focus to the vertex.

Identify the focus. For the function $y = \frac{1}{24}x^2$, $a = \frac{1}{24}$.

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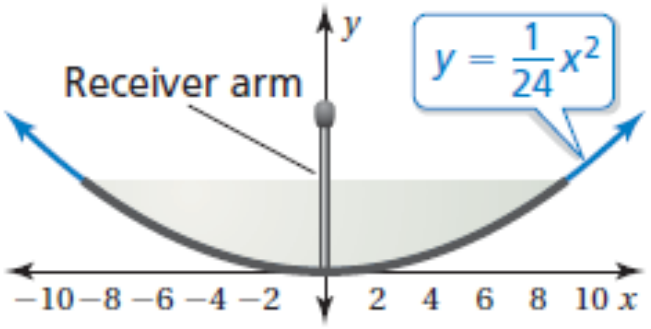

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$$\frac{1}{4a} = \frac{1}{4\left(\frac{1}{24}\right)}$$

Substitute $\frac{1}{24}$ for a.

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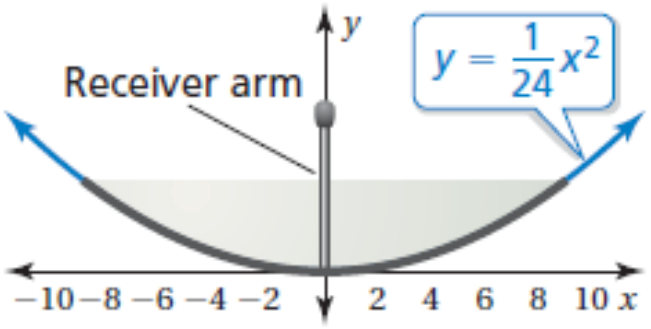

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$$\frac{1}{4a} = \frac{1}{4\left(\frac{1}{24}\right)}$$ Substitute $\frac{1}{24}$ for a.

$$= \frac{1}{\frac{1}{6}}$$ Multiply.

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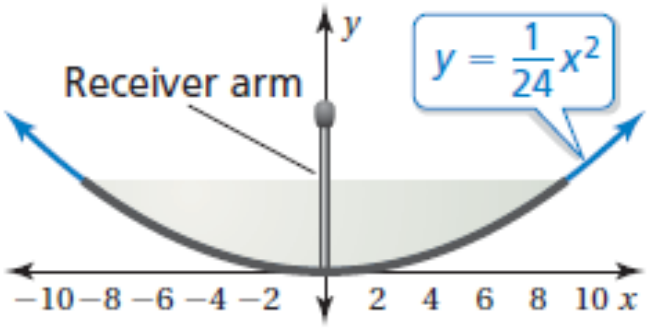

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$$= 6$$ Divide.

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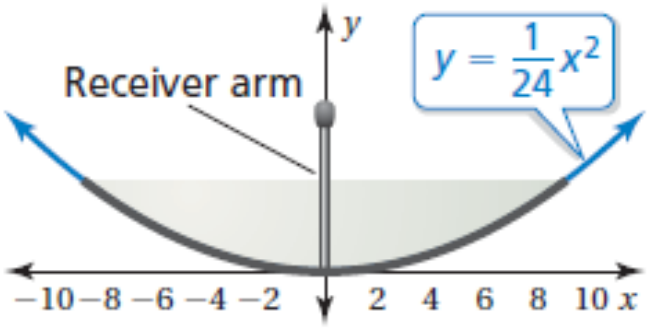

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$$= \frac{1}{\frac{1}{6}}$$ Multiply.

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The focus is (0,6). The vertex is (0, 0).
The distance from(0, 0) to (0, 6) is 6 units.