QUADRATIC FUNCTIONS

Lesson 8.1

“Linear” functions look like: $y = mx + b$

“Exponential” functions look like: $y = ab^x$

“Quadratic” functions look like: $y = ax^2 + bx + c$

Examples: $y = 2x^2 + 3x + 4$

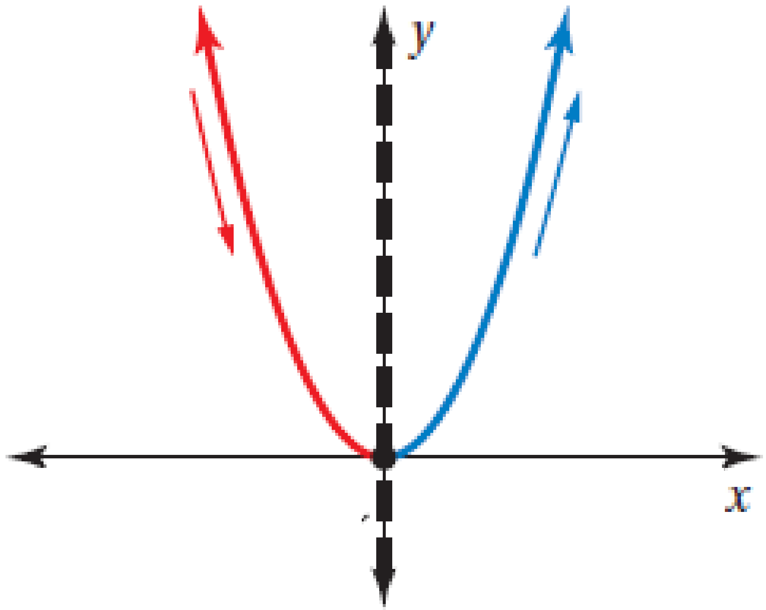
$y = -x^2 + 2x - 3$

A **quadratic function** is a nonlinear function that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$. The U-shaped graph of a quadratic function is called a **parabola.**

Key Idea

Characteristics of Quadratic Functions

The most basic quadratic function is $y = x^2$.

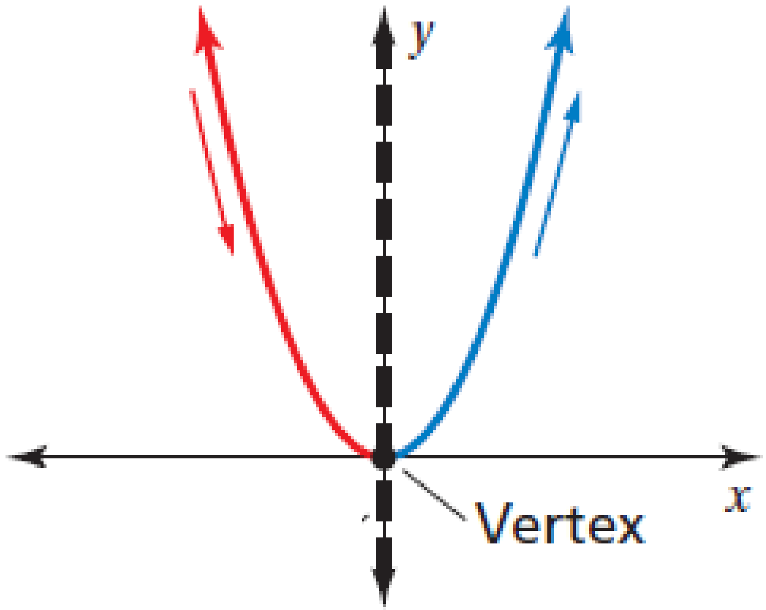


Key Idea

Characteristics of Quadratic Functions

The most basic quadratic function is $y = x^2$.

The lowest or highest point on a parabola is the **vertex.**



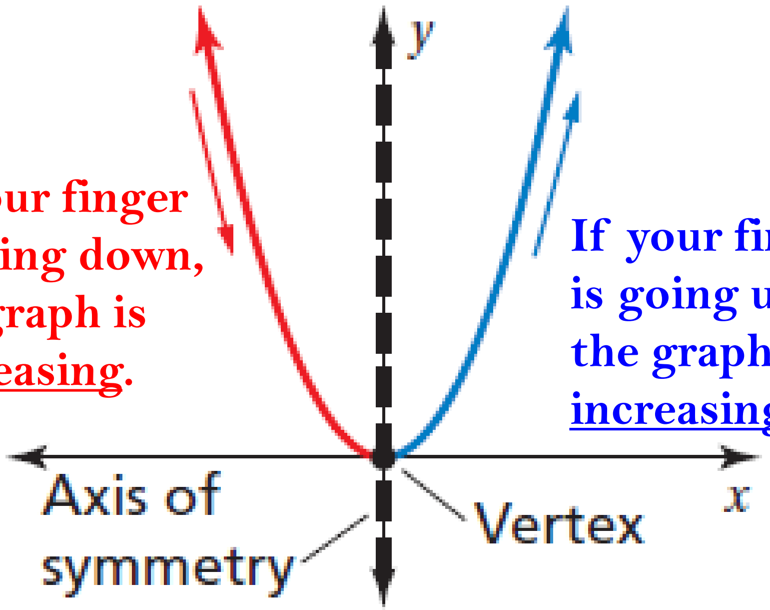
Key Idea

Characteristics of Quadratic Functions

The most basic quadratic function is $y = x^2$.

The lowest or highest point on a parabola is the **vertex.**

If your finger is going down, the graph is <u>decreasing</u>.



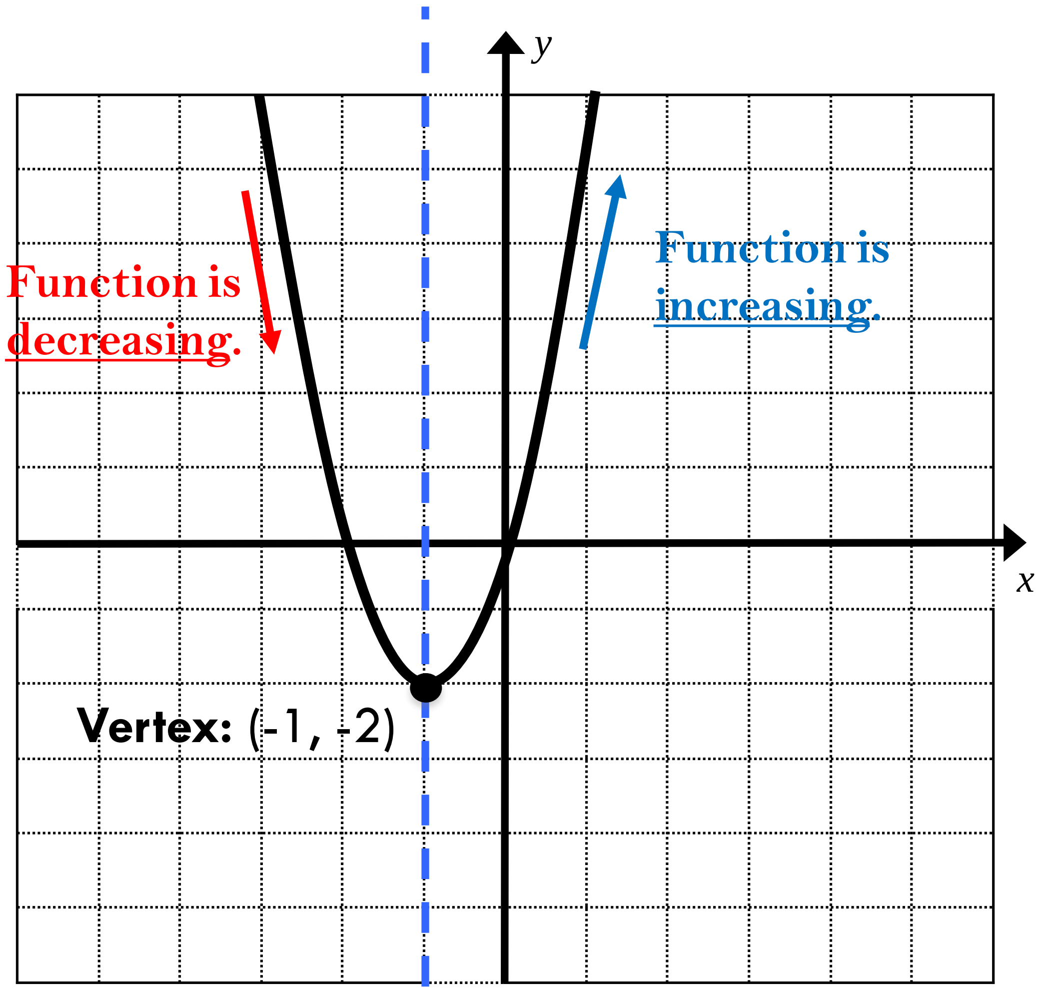
If your finger is going up, the graph is <u>increasing</u>.

The vertical line that divides the parabola into two symmetric parts is the **axis of symmetry.** The axis of symmetry passes through the vertex.

EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.

Using the graph, you can identify the vertex, axis of symmetry, and the behavior of the graph shown.



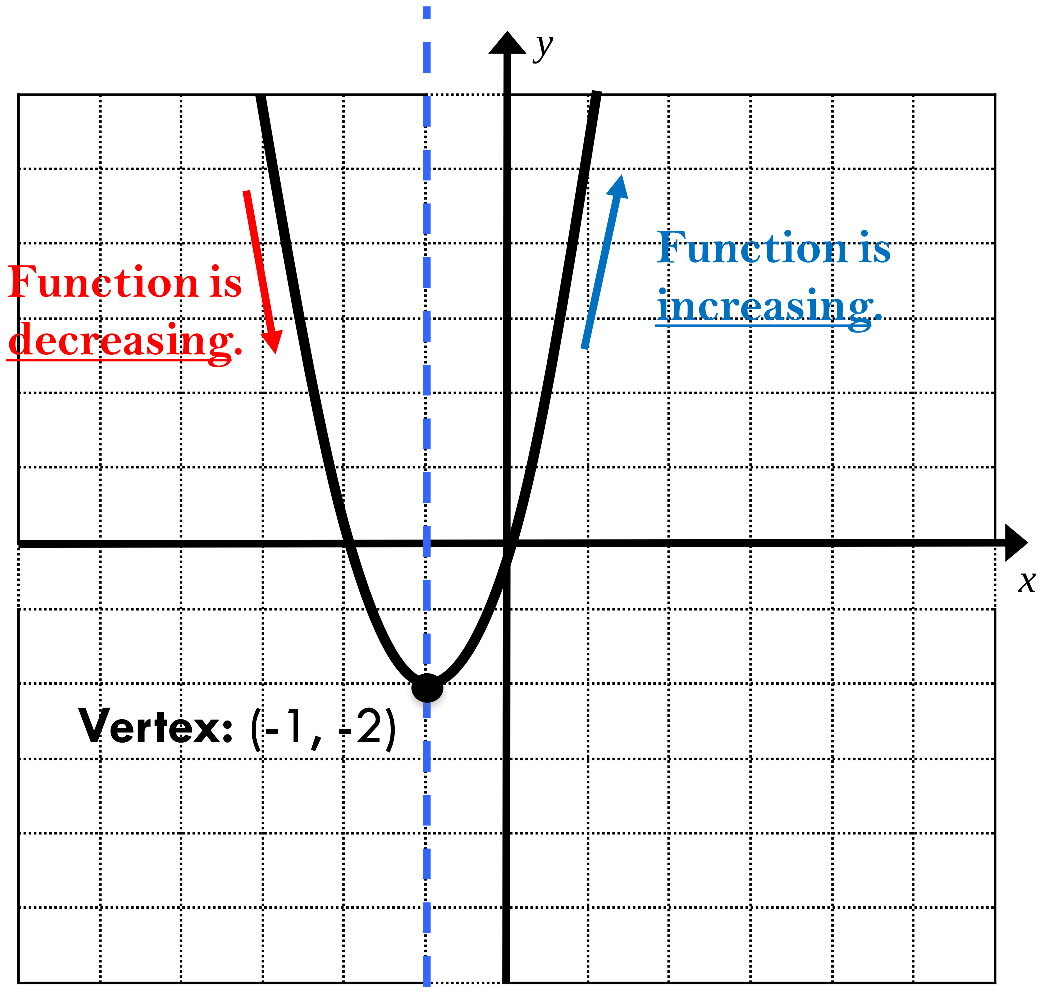
Axis of Symmetry: $x = -1$

EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.

Using the graph, you can identify the vertex, axis of symmetry, and the behavior of the graph shown.

You can also determine the following:



Axis of Symmetry: $x = -1$

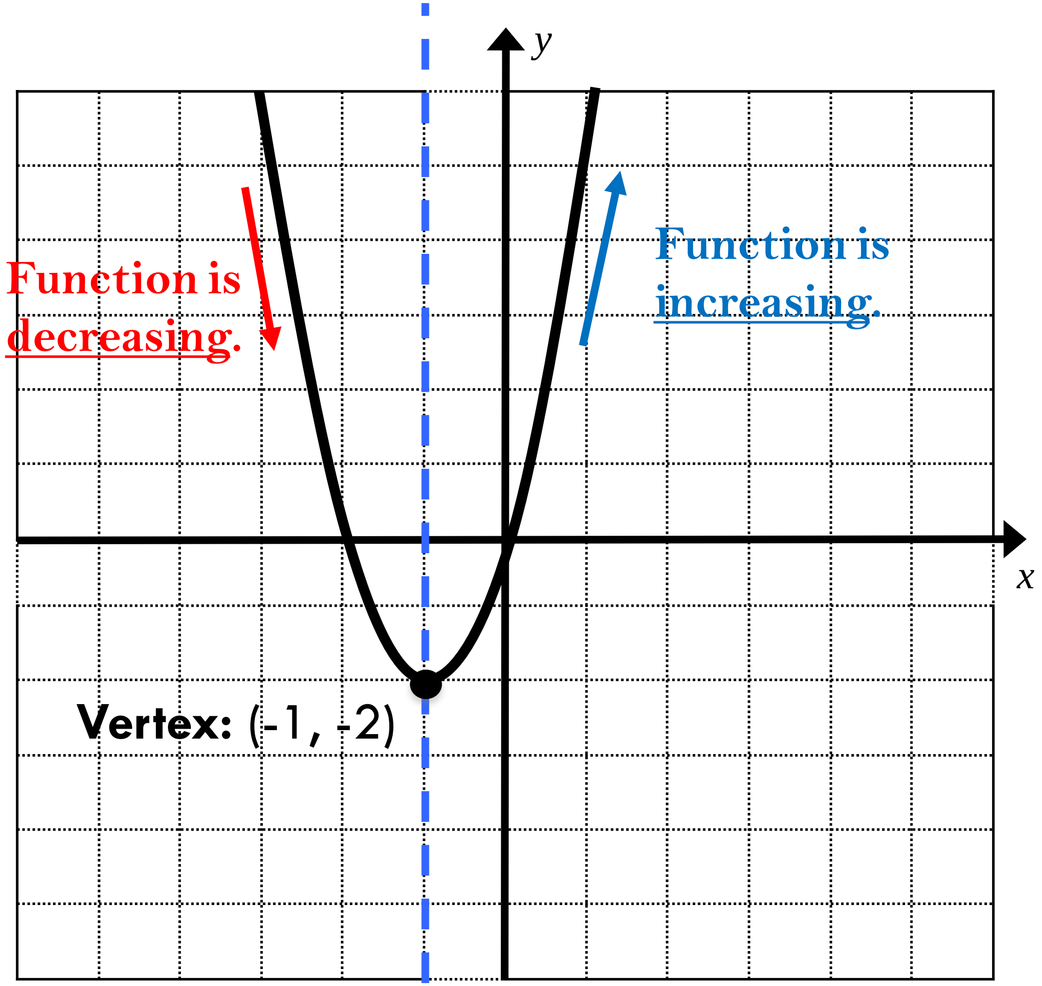
EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.

Using the graph, you can identify the vertex, axis of symmetry, and the behavior of the graph shown.

You can also determine the following:

- The domain is all real numbers.



Axis of Symmetry: $x = -1$

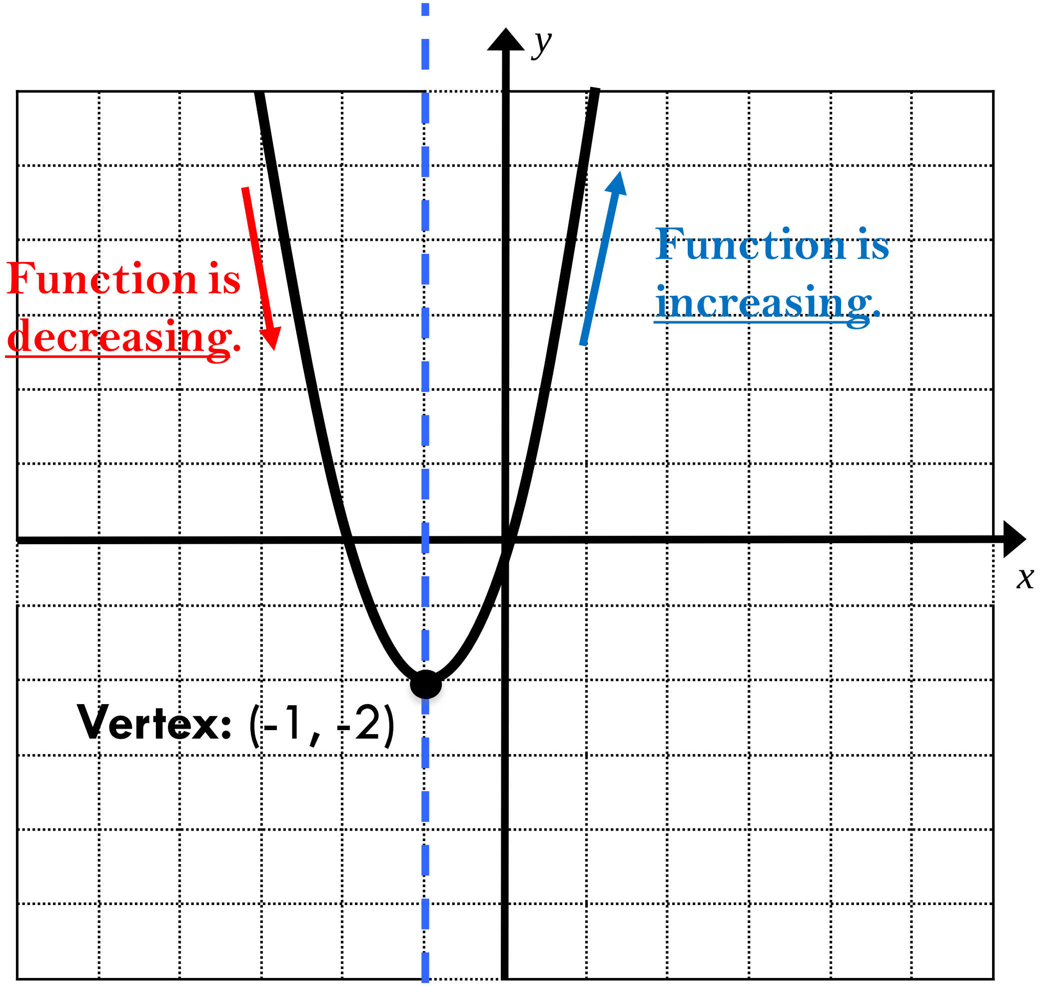
EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.

Using the graph, you can identify the vertex, axis of symmetry, and the behavior of the graph shown.

You can also determine the following:

- The domain is all real numbers.
- The range is all real numbers greater than or equal to -2.



Axis of Symmetry: $x = -1$

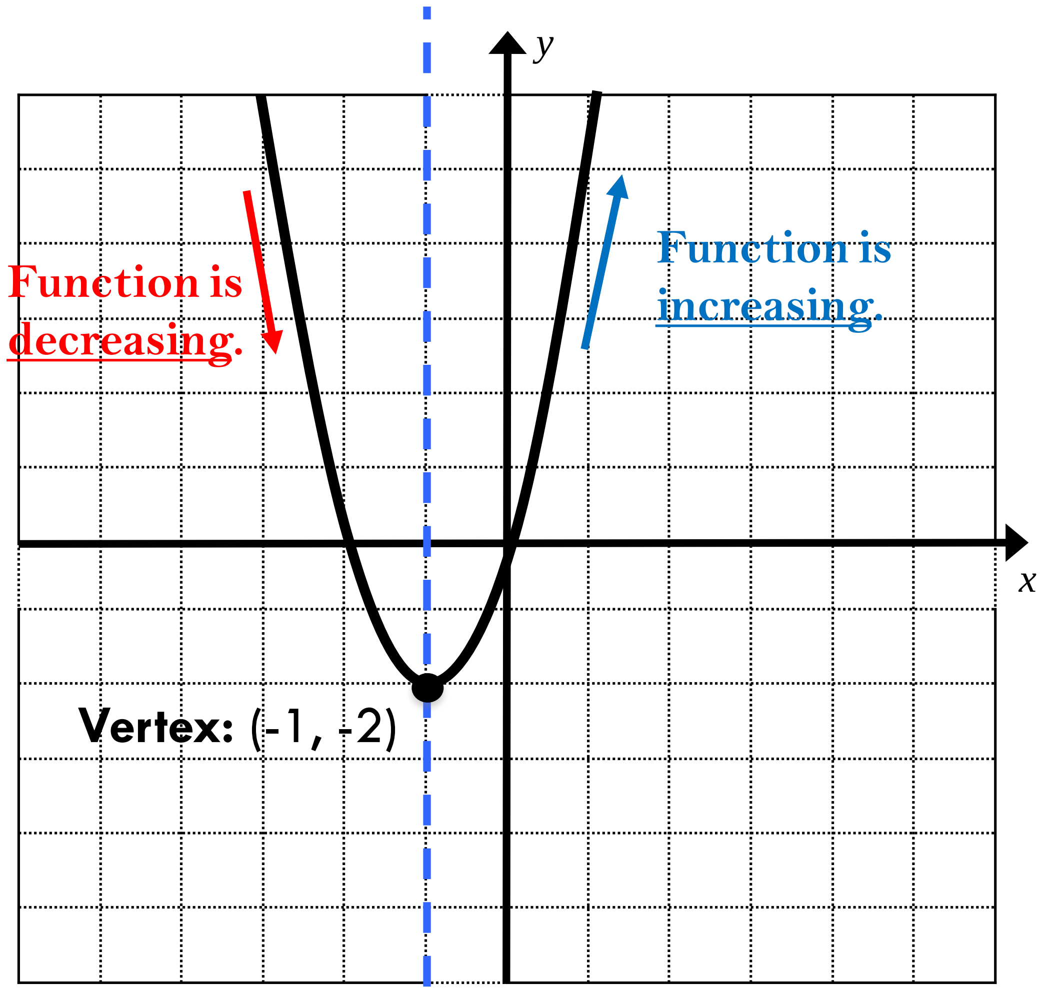
EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.

Using the graph, you can identify the vertex, axis of symmetry, and the behavior of the graph shown.

You can also determine the following:

- The domain is all real numbers.
- The range is all real numbers greater than or equal to -2.
- When $x < -1$, y increases as x decreases.



Axis of Symmetry: $x = -1$

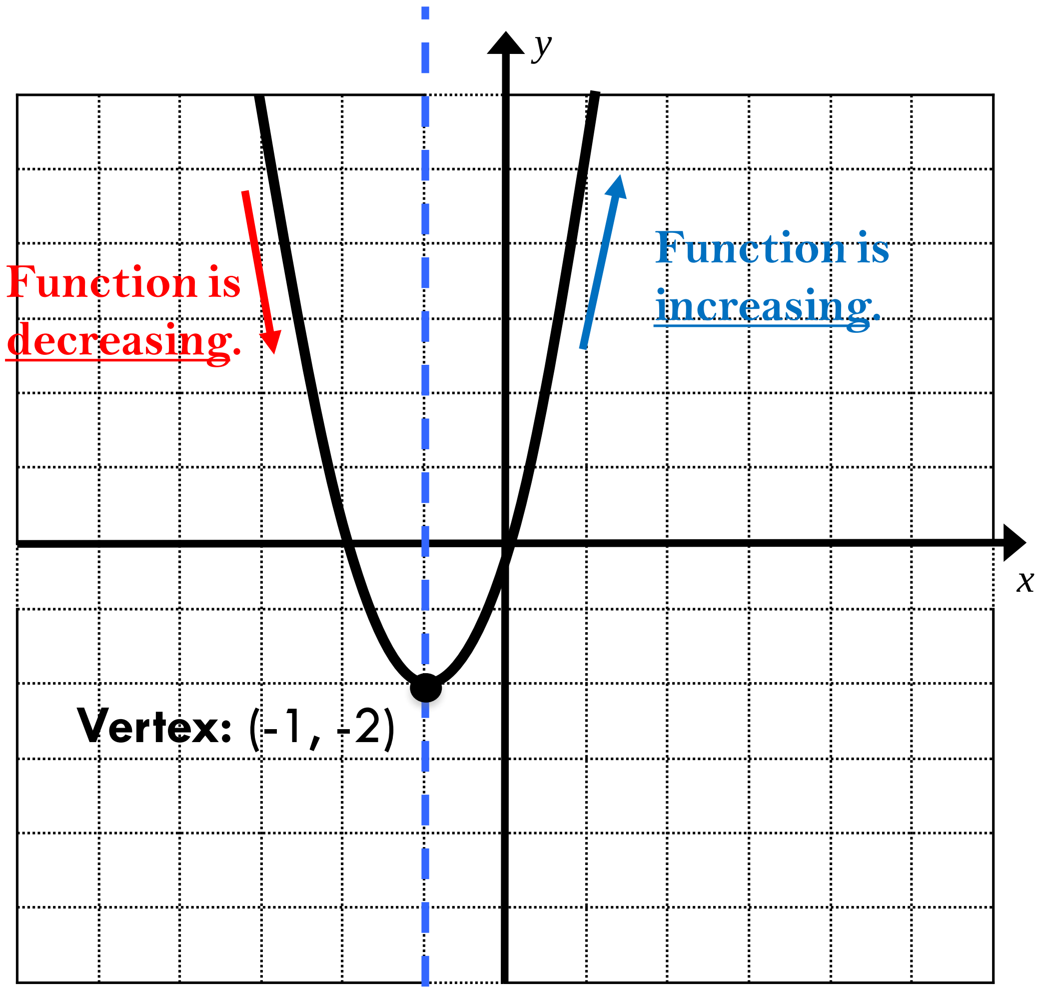
EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.

Using the graph, you can identify the vertex, axis of symmetry, and the behavior of the graph shown.

You can also determine the following:

- The domain is all real numbers.
- The range is all real numbers greater than or equal to -2.
- When $x < -1$, y increases as x decreases.
- When $x > -1$, y increases as x increases.

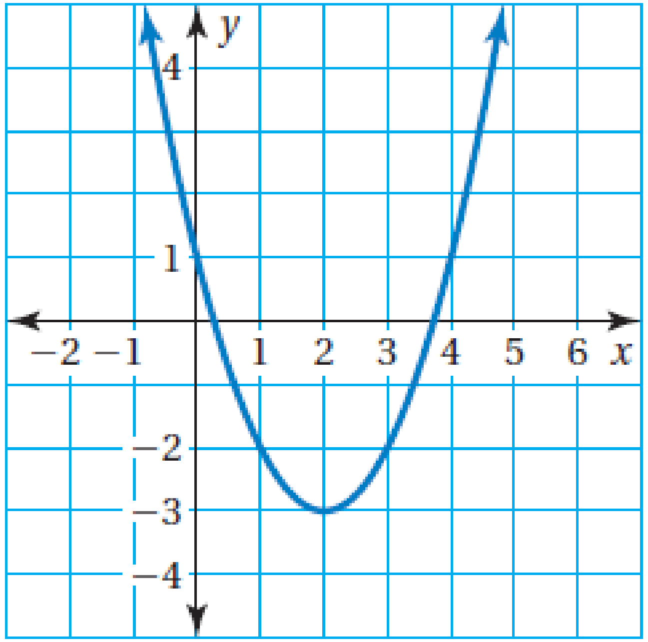


Axis of Symmetry: $x = -1$

On Your Own

Identify characteristics of the graph of the quadratic function.

1.



Vertex: (2, -3)

Axis of Symmetry: $x = 2$

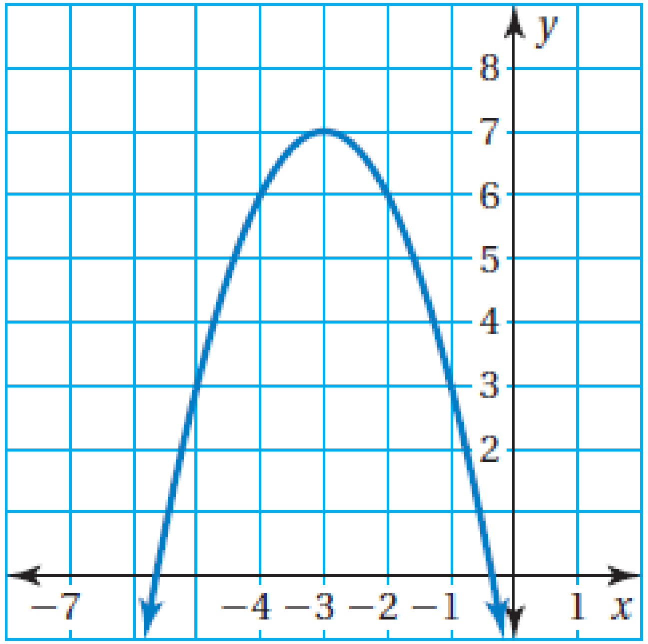
Domain: All real #'s

Range: $y \geq -3$

When $x < 2, y$ increases as x decreases

When $x > 2, y$ increases as x increases

2.



Vertex: (-3, 7)

Axis of Symmetry: $x = -3$

Domain: All real #'s

Range: $y \leq 7$

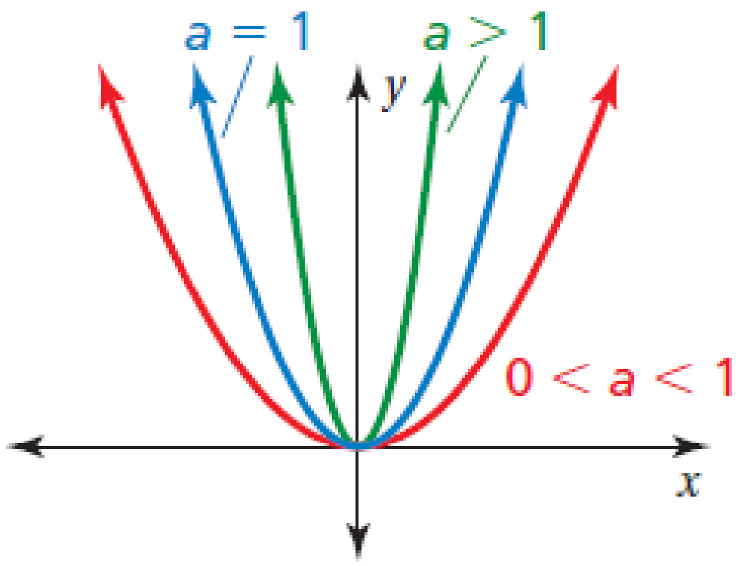
When $x < -3, y$ decreases as x decreases

When $x > -3, y$ decreases as x increases

Key Ideas

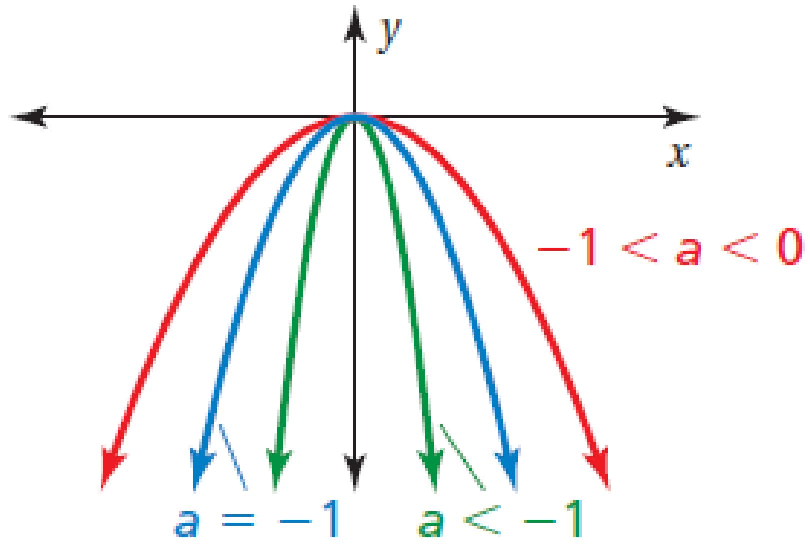
Graphing $y = ax^2$ When $a > 0$

- When $0 < a < 1$, the graph of $y = ax^2$ opens up and is wider than the graph of $y = x^2$.
- When $a > 1$, the graph of $y = ax^2$ opens up and is narrower than the graph of $y = x^2$.



Graphing $y = ax^2$ When $a < 0$

- When $-1 < a < 0$, the graph of $y = ax^2$ opens down and is wider than the graph of $y = x^2$.
- When $a < -1$, the graph of $y = ax^2$ opens down and is narrower than the graph of $y = x^2$.

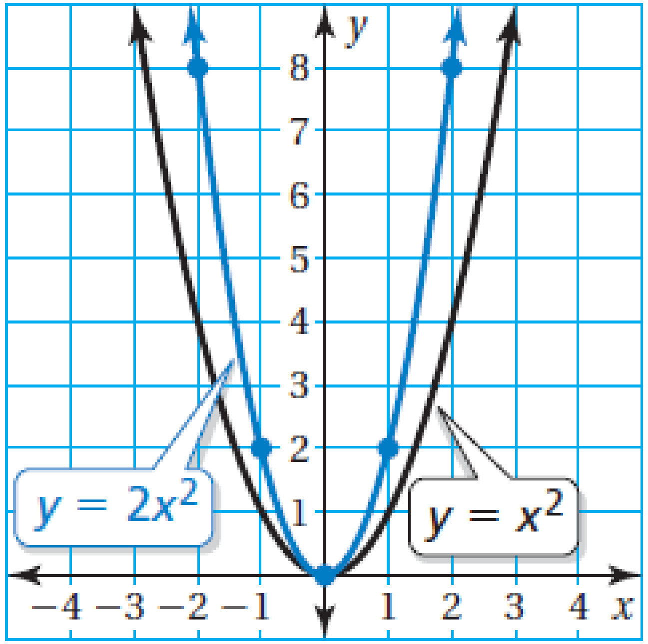


EXAMPLE 2 Graphing $y = ax^2$ When $a > 0$

Graph $y = 2x^2$. Compare the graph to the graph of $y = x^2$.

Make a table of values.

x	-2	-1	0	1	2
y	8	2	0	2	8



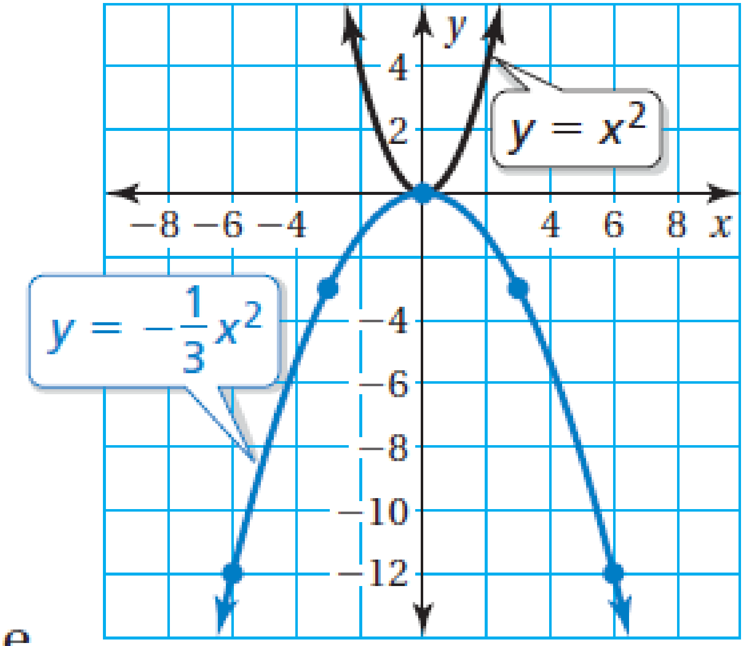
Both graphs open up and have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$.

EXAMPLE 3 Graphing $y = ax^2$ When $a < 0$

Graph $y = -\frac{1}{3}x^2$. Compare the graph to the graph of $y = x^2$.

Make a table of values. Choose x-values that make the calculations simple.

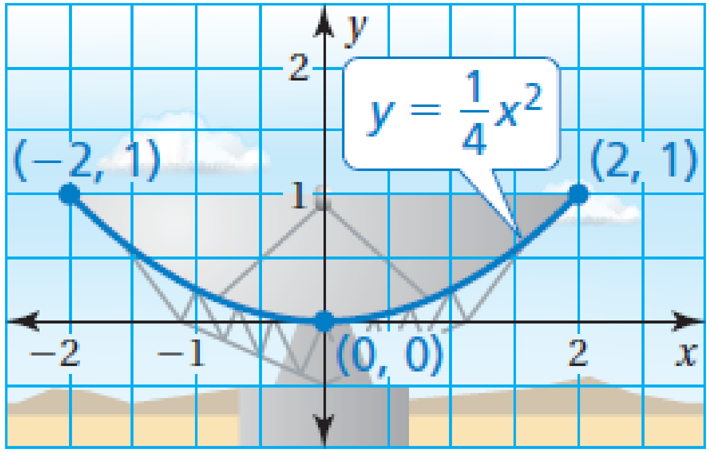
x	-6	-3	0	3	6
y	-12	-3	0	-3	-12



The graphs have the same vertex, (0, 0), and the same axis of symmetry, $x = 0$, but the graph of $y = -\frac{1}{3}x^2$ opens down. The graph of $y = -\frac{1}{3}x^2$ is wider than the graph of $y = x^2$.

EXAMPLE 4 Real-Life Application

The diagram shows the cross section of a satellite dish, where x and y are measured in meters. Find the width and depth of the dish.



Use the domain of the function to find the width of the dish. Use the range to find the depth.

The leftmost point on the graph is $(-2, 1)$ and the rightmost point is $(2, 1)$. So, the domain is $-2 \le x \le 2$, which represents 4 meters.

The lowest point on the graph is $(0, 0)$ and the highest points on the graph are $(-2, 1)$ and $(2, 1)$. So, the range is $0 \le y \le 1$, which represents 1 meter.

So, the satellite dish is 4 meters wide and 1 meter deep.