## Factoring Trinomials <br> $x^{2}+b x+c$

## Lesson 7.7

Essential Question how can you factor the trinomial $x^{2}+b x+c$ into the product of two binomials?

Consider the polynomial $x^{2}+b x+c$, where $b$ and $c$ are integers. To factor this polynomial as $(x+p)(x+q)$, you need to find integers $p$ and $q$ such that $p+q=b$ and $p q=c$.

## GO Key Idea

Factoring $x^{2}+b x+c$ When $c$ Is Positive
Algebra $\quad x^{2}+b x+c=(x+p)(x+q)$ when $p+q=b$ and $p q=c$. When $c$ is positive, $p$ and $q$ have the same sign as $b$.

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Examples $x^{2}+6 x+5=(x+1)(x+5)$

$$
x^{2}-6 x+5=(x-1)(x-5)
$$

Factor $x^{2}+10 x+16$
Notice that $b=10$ and $c=16$.
Find two factors of 16 that when multiplied together give the top number, and when added together give the bottom number.

The Magic X


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Find two factors of 16 that when multiplied together give the top number, and when added together give the bottom number.

Factors of 16 :

$$
\begin{array}{ll}
1,16 & =17 \\
2,8 & =10 \\
4,4 & =8
\end{array}
$$



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When $c$ is positive, $p$ and $q$ have the same sign as $b$.
Factored Form: $(x+2)(x+8)$

## On Your Own

## Factor the polynomial.

1. $x^{2}+2 x+1=(x+1)(x+1)$

2. $y^{2}+6 y+8=(y+4)(y+2)$

3. $x^{2}+9 x+8=(x+8)(x+1)$

4. $z^{2}+11 z+24=(z+8)(z+3)$


## EXAMPLE 2 Factoring $x^{2}+b x+c$ When $b$ Is Negative and $c$ Is Positive

Factor $x^{2}-8 x+12$.
Notice that $b=-8$ and $c=12$.
The Magic X
Find two negative factors of 12 that when multiplied together equals positive 12, and when added equals negative 8 .

## Factors of 12:

$$
\begin{aligned}
-1,-12 & =-13 \\
-2,-6 & =-8 \\
-3,-4 & =-7
\end{aligned}
$$



When $c$ is positive, $p$ and $q$ have the same sign as $b$.

$$
\text { Factored Form: }(x-2)(x-6)
$$

## On Your Own

Factor the polynomial.
5. $w^{2}-4 w+3=(w-3)(w-1)$

7. $x^{2}-14 x+24=(x-2)(x-12)$

6. $n^{2}-12 n+35=(n-7)(n-5)$


## Key Idea

Factoring $x^{2}+b x-c$
If the middle term is positive and the last term is negative, then one factor is negative and the other factor is positive. The larger number must be positive.
Example $\quad x^{2}+4 x-5$
The Magic X


$$
=(x+5)(x-1)
$$

## Key Idea

Factoring $x^{2}-b x-c$
If the middle term is negative and the last term is negative, then one factor is negative and the other factor is positive. The larger number must be negative.
Example $\quad x^{2}-13 x-48$
The Magic X


$$
=(x-16)(x+3)
$$

## On Your Own

Factor the polynomial.

10. $z^{2}-z-12=(z-4)(z+3)$

9. $y^{2}+13 y-30=(y+15)(y-2)$

11. $m^{2}-11 m-26=(m-13)(m+2)$


## Real-Life Application



A farmer plants a rectangular pumpkin patch in the northeast corner of the square plot of land. The area of the pumpkin patch is 600 square meters. What is the area of the square plot of land?

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600=(s-30)(s-40)
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\begin{aligned}
& 600=(s-30)(s-40) \\
& 600=s^{2}-70 s+1200
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0=s^{2}-70 s+600 \\
0=(s-10)(s-60) \\
s-10=0 \quad \text { or } \quad s-60=0
\end{gathered}
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The diagram shows that the side length is at least 30 meters, so 10 meters does not make sense in this situation. The width is 60 meters.

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The diagram shows that the side length is at least 30 meters, so 10 meters does not make sense in this situation. The width is 60 meters.

So the area of the plot of land is $60(60)=3600$ square meters.

## Summary

Factoring $x^{2}+b x+c$ as $(x+p)(x+q)$
The diagram shows the relationships between the signs of $b$ and $c$ and the signs of $p$ and $q$.


