

Essential Question How can you factor the trinomial $x^2 + bx + c$ into the product of two binomials?

Consider the polynomial $x^2 + bx + c$, where *b* and *c* are integers. To factor this polynomial as (x + p)(x + q), you need to find integers *p* and *q* such that p + q = b and pq = c.

🕝 Key Idea

Factoring $x^2 + bx + c$ When *c* Is Positive Algebra $x^2 + bx + c = (x + p)(x + q)$ when p + q = b and pq = c. When *c* is positive, *p* and *q* have the same sign as *b*.



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Factoring $x^2 + bx + c$ When b and c Are Positive

Factor $x^2 + 10x + 16$.

EXAMPLE

1

Notice that b = 10 and c = 16.

Find two factors of 16 that when multiplied together give the top number, and when added together give the bottom number.

The Magic X





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Factors of 16:

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EXAMPLE

$$\begin{array}{rcl}
1,16 &= 17 \\
2,8 &= 10 \\
4,4 &= 8
\end{array}$$

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When *c* is positive, *p* and *q* have the same sign as *b*.

Factored Form: (x + 2)(x + 8)





Factor the polynomial.

1.
$$x^2 + 2x + 1 = (x + 1)(x + 1)$$





2. $x^2 + 9x + 8 = (x + 8)(x + 1)$

4. $z^2 + 11z + 24 = (z + 8)(z + 3)$







Factoring $x^2 + bx + c$ When b is Negative and c is Positive

Factor $x^2 - 8x + 12$.

Notice that b = -8 and c = 12.

The Magic X

Find two negative factors of 12 that when multiplied together equals positive 12, and when added equals negative 8.

Factors of 12:

$$\begin{array}{rcl} -1, -12 &= -13 \\ \hline -2, -6 &= -8 \\ \hline -3, -4 &= -7 \end{array}$$



When *c* is positive, *p* and *q* have the same sign as *b*.

Factored Form: (x-2)(x-6)





Factor the polynomial.

5.
$$w^2 - 4w + 3 = (w - 3)(w - 1)$$



6.
$$n^2 - 12n + 35 = (n - 7)(n - 5)$$

7.
$$x^2 - 14x + 24 = (x - 2)(x - 12)$$





Factoring $x^2 + bx - c$

If the middle term is positive and the last term is negative, then one factor is negative and the other factor is positive. **The larger number must be positive.**

Example
$$x^2 + 4x - 5$$

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$$= (x+5)(x-1)$$





Factoring $x^2 - bx - c$

If the middle term is negative and the last term is negative, then one factor is negative and the other factor is positive. **The larger number must be negative.**

Example
$$x^2 - 13x - 48$$

The Magic X



$$= (x - 16)(x + 3)$$





Factor the polynomial.

8.
$$x^2 + 2x - 15 = (x + 5)(x - 3)$$

10.
$$z^2 - z - 12 = (z - 4)(z + 3)$$

9.
$$y^{2} + 13y - 30 = (y + 15)(y - 2)$$

 $15 - 2$
 $15 - 2$
 $11. m^{2} - 11m - 26 = (m - 13)(m + 2)$
 $-13 - 26$

-11





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 $600 = s^2 - 70s + 1200$





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0 = (s - 10)(s - 60)





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So the area of the plot of land is 60(60) = 3600 square meters.





Factoring $x^2 + bx + c$ as (x + p)(x + q)

The diagram shows the relationships between the signs of *b* and *c* and the signs of *p* and *q*.



