Solving Polynomials in Factored Form

Lesson 7.5

Essential Question

How can you solve a polynomial equation that is written in factored form?

Equivalent Forms of Polynomial Equations.

A polynomial is in **factored form** when it is written as a product of two or more factors.

Standard form	***Factored form***
$x^2 + 2x$	$x(x + 2)$
$x^2 + 5x - 24$	$(x - 3)(x + 8)$
$r^3 + 8r^2 - 16r - 128$	$(r - 4)(r + 4)(r + 8)$

When one side of an equation is a polynomial in factored form and the other side is 0, use the **Zero-Product Property** to solve the polynomial equation. The solutions of the polynomial equations are also called **roots**.

Key Idea

Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

Example 1 Solving Polynomial Equations

a. $(x)(x + 7) = 0$

$x = 0$ or $x + 7 = 0$

$x = -7$

The roots are $x = 0$ *or* $x = -7$.

b. $(x - 4)(x + 3) = 0$

$x - 4 = 0$ or $x + 3 = 0$

$x = 4$ or $x = -3$

The roots are $x = 4$ *or* $x = -3$.

On Your Own

Solve the equation.

1. $(x)(x-1)=0$

   $x=0 \ \ or \ \ x=1$

2. $3t(t+2)=0$

   $t=0 \ \ or \ \ t=-2$

3. $(z-4)(z+4)(z-2)=0$

   $z=4 \ \ or \ \ z=-4 \ \ or \ \ z=2$

4. $(b+7)^2=0$

   $b=-7$

Example 2 Solving Polynomial Equation

a. $(3n+8)(3n-8)=0$

$3n+8=0 \quad or\ 3n-8=0$

$3n=-8 \quad or\ 3n=8$

$n=-\frac{8}{3} \quad or\ n=\frac{8}{3}$

b. $(2-4x)^2$

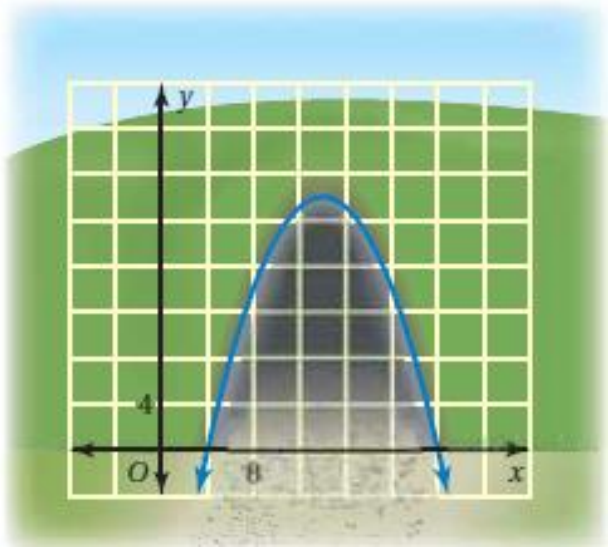
$(2-4x)(2-4x)$

$2-4x=0$

$-4x=-2$

$x=\frac{1}{2}$

Real-Life Application



Choose Tools The entrance of a tunnel can be modeled by $y = \frac{11}{50}(x-4)(x-24)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the tunnel at the ground level.

$$0 = \frac{11}{50}(x-4)(x-24)$$

$$\left(\frac{50}{11}\right) 0 = \cancel{\frac{11}{50}}(x-4)(x-24) \cancel{\left(\frac{50}{11}\right)}$$

$$0 = (x-4)(x-24)$$

$$x - 4 = 0 \; or \; x - 24 = 0$$

$$x = 4 \quad \text{or} \quad x = 24$$

The width is the distance between the x-coordinates, 4 and 24.

So the width of the tunnel is $24 - 4 = 20 \; feet$

On Your Own

Solve the equation.

5. $(3p + 5)(3p - 5) = 0$

$p = -\frac{5}{3}$ *or* $p = \frac{5}{3}$

6. $(12 - 6x)^2 = 0$

$x = 2$

7. The entrance of a mine shaft can be modeled by $y = -\frac{1}{2}(x - 4)(x + 4)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the entrance at the ground level.

$x = 4$ *or* $x = -4$

The width is the distance between the x-coordinates, 4 and -4.

So the width of the tunnel is $4 - (-4) = 8\ feet$