Solving Rational Equations

11.7

Solving Rational Equations
Method 1: Cross Products

A **rational equation** is an equation that contains rational expressions.

- Use Cross Products when each side of a rational equation consists of one rational expression.

  - Example: Solve $\dfrac{5}{x+4} = \dfrac{4}{x-4}$.

$$\dfrac{5}{x+4} = \dfrac{4}{x-4}$$

$$5(x-4) = 4(x+4)$$

$$5x - 20 = 4x + 16$$

$$5x = 4x + 36$$

$$x = 36$$

Check

$$\dfrac{5}{x+4} = \dfrac{4}{x-4}$$

$$\dfrac{5}{36+4} \overset{?}{=} \dfrac{4}{36-4}$$

$$\dfrac{1}{8} = \dfrac{1}{8} \checkmark$$

On Your Own

Solve the equation. Check your solution(s).

1. $\dfrac{2}{x-3} = \dfrac{4}{x-7}$ $\qquad x = -1$

2. $\dfrac{4}{z+4} = \dfrac{z}{z+1}$ $\qquad z = -2, z = 2$

3. $\dfrac{3y}{4} = \dfrac{6}{y+7}$ $\qquad y = -8, y = 1$

Solving Rational Equations Method 2: Multiply by LCD

- When there is more than one rational expression on one or both sides of a rational equation, multiply each side by the LCD and then solve.

- This will get rid of all the fractions.
  - Example: **Solve** $\dfrac{z}{z-4} - \dfrac{3}{4} = \dfrac{3}{z-4}$
  - LCD: $4(z-4)$

$$4(z-4)\left(\frac{z}{z-4}\right) - \left(\frac{3}{4}\right)4(z-4) = 4(z-4)\left(\frac{3}{z-4}\right)$$

Check: $\dfrac{0}{0-4} - \dfrac{3}{4} = \dfrac{3}{0-4}$

$$4(z) - 3(z-4) = 4(3)$$

$$0 - \frac{3}{4} = \frac{3}{-4}$$

$$4z - 3z + 12 = 12$$

$$-\frac{3}{4} = -\frac{3}{4}$$

$$z + 12 = 12$$

$$z = 0$$

Extraneous Solutions

- Beware of Extraneous Solutions.
  - Example: **Solve** $\dfrac{z}{z-2} - \dfrac{2}{3} = \dfrac{2}{z-2}$
  - LCD: $3(z-2)$

$$3(z-2)\left(\frac{z}{z-2}\right) - \left(\frac{2}{3}\right)3(z-2) = 3(z-2)\left(\frac{2}{z-2}\right)$$

$$3(z) - 2(z-2) = 3(2)$$

$$3z - 2z + 4 = 6$$

$$z + 4 = 6$$

$$z = 2$$

Because each side of the equation is undefined when $z = 2$, it is an extraneous solution. So, the equation has **no solution**.

Real Life Application

Anne and Maria play tennis almost every weekend. So far, Anne has won 12 out of 20 matches.

How many matches will Anne have to win in a row to improve her winning percentage to 75%?

$$\frac{12 + m}{20 + m} = 0.75$$

$$(20 + m)\frac{12 + m}{20 + m} = 0.75(20 + m)$$

$$12 + m = 15 + 0.75m$$

$$0.25m = 3$$

$$m = 12$$

Anne will need to win 12 matches in a row to improve her winning percentage to 75%.