Adding and Subtracting Rational Expressions

Lesson 11.6

Adding and Subtracting Rational Expressions w/ Like Denominators

- To add or subtract fractions with like denominators, simply add or subtract the numerators.

- Write the sum or difference over the common denominator.

- Let $a, b, and\ c$ be polynomials, $where\ c \neq 0.$

- Adding: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}.$

- Subtracting: $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}.$

Ex. 1: Find $\dfrac{3}{x+2} + \dfrac{1}{x+2}$.

$$\dfrac{3}{x+2} + \dfrac{1}{x+2} = \dfrac{3+1}{x+2}$$

Since x + 2 is the common denominator, add the numerators.

$$= \dfrac{4}{x+2}$$

Ex. 2: Find $\dfrac{3a+2}{a-7} - \dfrac{a-3}{a-7}.$

$$\dfrac{3a+2}{a-7} - \dfrac{a-3}{a-7} = \dfrac{(3a+2)-(a-3)}{a-7}$$

Add the additive inverse of a - 3.

$$= \dfrac{3a+2+(-a+3)}{a-7}$$

Combine like terms.

$$= \dfrac{2a+5}{a-7}$$

Remember when subtracting rational expressions to add the additive inverse of the second expression.

Express in Simplest Form

Find $\dfrac{2y}{y+1} - \dfrac{-2y-4}{y+1}$

$= \dfrac{2y-(-2y-4)}{y+1}$ Subtract the numerators.

$= \dfrac{2y+2y+4}{y+1}$ Use the Distributive Property.

$= \dfrac{4y+4}{y+1}$ Combine like terms.

$= \dfrac{4(y+1)}{y+1}$ Factor. Divide out the common factor.

$= 4$ Simplify.

On Your Own

Find the sum or difference.

1. $\dfrac{4}{9z} - \dfrac{8}{9z}$ $-\dfrac{4}{9z}$

2. $\dfrac{3w+1}{w-1} + \dfrac{w}{w-1}$ $\dfrac{4w+1}{w-1}$

3. $\dfrac{x+3}{x^2+x-2} - \dfrac{1}{x^2+x-2}$ $\dfrac{1}{x-1}$

Adding or Subtracting Rational Expressions w/ Unlike Denominators

- To add or subtract fractions with unlike denominators, find the Least Common Denominator (LCD).

- The LCD is the Least Common Multiple of the denominators.

Find the LCD of $\dfrac{3}{10g^2}$ **and** $\dfrac{5}{12g}$**.**

First write the prime factorization of each denominator.

$$10g^2 = 2 \cdot 5 \cdot g^2 \qquad\qquad 12g = 2^2 \cdot 3 \cdot g$$

Use the greatest power of each factor that appears in either denominator to find the LCM of the denominators.

$$\text{LCM} = 2^2 \cdot 3 \cdot 5 \cdot g^2 = 60g^2$$

On Your Own

Find the LCD of the rational expressions.

4. $\dfrac{2}{7g}, \ -\dfrac{15}{4g^3}$

$28g^3$

5. $\dfrac{8}{n}, \ \dfrac{n}{n+1}$

$n(n+1)$

6. $\dfrac{t}{t^2-4}, \ \dfrac{9}{t-2}$

$(t-2)(t+2)$

7. $\dfrac{x+1}{x^2-x-6}, \ \dfrac{5}{x(x-3)}$

$x(x-3)(x+2)$

Adding w/ Unlike Denominators

Find the sum $\dfrac{1}{8x} + \dfrac{x-2}{6x^2}$.

Because the expressions have unlike denominators, find the LCD.

$$8x = 2^3 \cdot x \qquad\qquad\qquad 6x^2 = 2 \cdot 3 \cdot x^2$$

The LCD is $2^3 \cdot 3 \cdot x^2 = 24x^2$.

$$\frac{1}{8x} + \frac{x-2}{6x^2} = \frac{1(3x)}{8x(3x)} + \frac{(x-2)(4)}{6x^2(4)} \qquad \text{Rewrite using the LCD, } 24x^2.$$

$$= \frac{3x}{24x^2} + \frac{4x-8}{24x^2} \qquad \text{Simplify.}$$

$$= \frac{3x + 4x - 8}{24x^2} \qquad \text{Add the numerators.}$$

$$= \frac{7x - 8}{24x^2} \qquad \text{Simplify.}$$

Subtracting w/ Unlike Denominators

$$\frac{x+3}{x^2-8x+12} - \frac{2}{x-6} = \frac{x+3}{(x-6)(x-2)} - \frac{2}{x-6}$$

Factor $x^2 - 8x + 12$.

$$= \frac{x+3}{(x-6)(x-2)} - \frac{2(x-2)}{(x-6)(x-2)}$$

Rewrite using the LCD, $(x-6)(x-2)$.

$$= \frac{(x+3) - 2(x-2)}{(x-6)(x-2)}$$

Subtract the numerators.

$$= \frac{-x+7}{(x-6)(x-2)}$$

Simplify.

Real Life Application

You row your kayak 6 miles downstream from your campsite to a dam, and then you row back to your campsite. You row x miles per hour during the entire trip, and the river current is 2 miles per hour. Write an expression for the total time of the trip.

Solving the formula $d = rt$ for time t gives $t = \dfrac{d}{r}$. Use this to write an expression for the total time of the trip.

Time downstream			**Time upstream**	
Distance downstream	÷	Speed downstream	Distance upstream	÷ Speed upstream
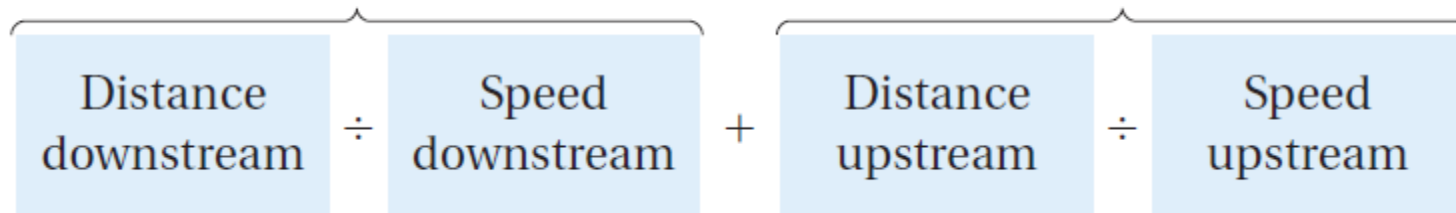
$$\frac{6}{x+2} + \frac{6}{x-2} \;=\; \frac{6(x-2)}{(x+2)(x-2)} + \frac{6(x+2)}{(x-2)(x+2)}$$

Real Life Application

Time downstream

Time upstream

| Distance downstream | ÷ | Speed downstream |
| Distance upstream | ÷ | Speed upstream |

$$\frac{6}{x+2} + \frac{6}{x-2} = \frac{6(x-2)}{(x+2)(x-2)} + \frac{6(x+2)}{(x-2)(x+2)}$$

$$= \frac{6(x-2) + 6(x+2)}{(x+2)(x-2)}$$

$$= \frac{6x - 12 + 6x + 12}{(x+2)(x-2)}$$

$$= \frac{12x}{(x+2)(x-2)}$$

On Your Own

Find the sum or difference.

8. $\dfrac{x+5}{3x^3} - \dfrac{2}{9x^2}$ $\dfrac{x+15}{9x^3}$

9. $\dfrac{2k}{k+1} + \dfrac{k}{k-2}$ $\dfrac{3k(k-1)}{(k+1)(k-2)}$

10. $\dfrac{3y-1}{y^2-64} - \dfrac{3}{y+8}$ $\dfrac{23}{(y+8)(y-8)}$