



Simplifying Rational Expressions

11.3

Rational Expressions

- A **rational expression** is a quotient of polynomials.
- Examples: $\frac{x-3}{5x+1}$, $\frac{3x^2+6x+7}{2x-6}$, $\frac{3q}{4q^4-4q-12}$
- Values that make the denominator zero are *excluded values* and the rational expressions are considered to be undefined at those values.



Simplifying Rational Expressions

- A **rational expression** is in simplest form when the numerator and denominator have no common factors except for 1.
- To simplify a rational expression, factor the numerator and denominator and *divide out* any common factors.
- Find *excluded* values using original expression or when original denominator is in factored form.
- Example: $\frac{2x+2}{5x+5} = \frac{2\cancel{(x+1)}}{5\cancel{(x+1)}} = \frac{2}{5}$; The excluded value is $x = -1$



Finding Excluded Values

Example: $\frac{2ab(a - 2)(b + 3)}{3ab(a^2 - 4)}$

Completely factor the original denominator.

$$\begin{aligned} & 3ab(a^2 - 4) \\ &= 3ab(a + 2)(a - 2) \end{aligned}$$

The excluded values are $a = 0, 2,$ and $- 2$ and $b = 0$.



Simplifying Rational Expressions

Simplify each rational expression, if possible. State the excluded value(s).

$$\text{a. } \frac{10}{2x^2} = \frac{2 \cdot 5}{2 \cdot x \cdot x} = \frac{\cancel{2} \cdot 5}{\cancel{2} \cdot x \cdot x}$$

$$= \frac{5}{x^2} \quad \text{The excluded value is } x = 0.$$

$$\text{b. } \frac{n}{2n + 8}$$

The expression is already in simplest form.
The excluded value is $n = -4$.

$$\text{c. } \frac{4y^2}{8y(y - 2)} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{y} \cdot y}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{y} (y - 2)}$$

$$= \frac{y}{2(y - 2)} \quad \text{The excluded values are } y = 0 \text{ and } y = 2.$$



Simplifying Rational Expressions

Simplify each rational expression, if possible. State the excluded value(s).

$$\text{d. } \frac{1 - y^2}{y - 1} = \frac{(1 - y)(1 + y)}{y - 1}$$

Difference of Two Squares Pattern

$$= \frac{-1(y - 1)(1 + y)}{y - 1}$$

Rewrite $1 - y$ as $-(y - 1)$

$$= \frac{-1\cancel{(y - 1)}(1 + y)}{\cancel{y - 1}}$$

Divide out the common factor.

$$= -y - 1$$

Simplify.

The excluded value is $y = 1$.



On Your Own

Simplify the rational expression, if possible. State the excluded value(s).

1. $\frac{2b + 8}{7b + 28}$

$\frac{2}{7}; b = -4$

2. $\frac{2a - 6}{4a^2 - 12a}$

$\frac{1}{2a}; a = 0, a = 3$

3. $\frac{z^2 - 6z - 16}{8 - z}$

$-z - 2; z = 8$

Real Life Application



$x + 3 \text{ in.}$

The side length of the square clock is represented by the expression $x + 3 \text{ in.}$ Write and simplify a rational expression for the ratio of the perimeter to the area.

$$\begin{aligned}\frac{\textit{Perimeter}}{\textit{Area}} &= \frac{4(x + 3)}{(x + 3)(x + 3)} \\ &= \frac{4}{(x + 3)}\end{aligned}$$