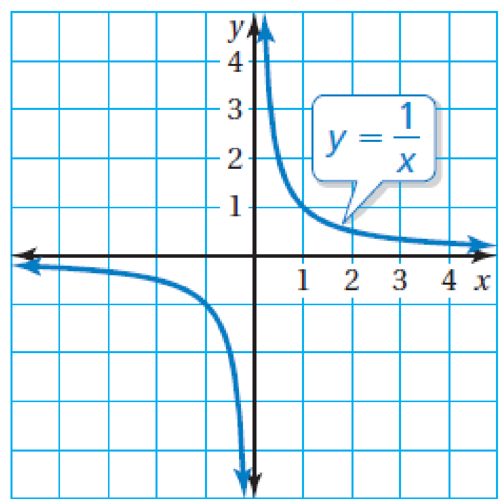
Graphing Rational Functions
11.2

The inverse variation equations in Section 11.1 are *rational functions*.

Key Idea

- A rational function is a function written in the form of $y = \dfrac{polynomial}{polynomial}$
- The denominator cannot equal 0.
- The most basic rational function or parent function is $y = \dfrac{1}{x}$.



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EXAMPLE 1 Finding the Excluded Value of a Rational Function

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$x = -5$	Subtract 5 from each side.

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$x = -5$ Subtract 5 from each side.

The excluded value is $x = -5$.

EXAMPLE 2 Graphing a Rational Function

Graph $y = \frac{1}{x-1}$. Describe the domain and range.

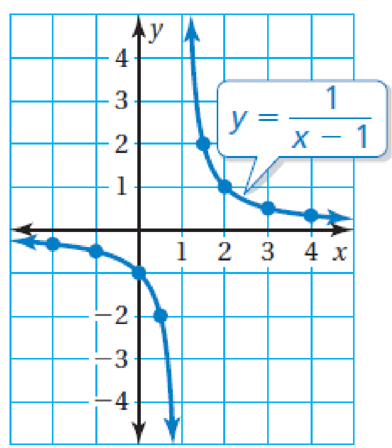
The excluded value is $x = 1$, so choose x-values on either side of 1.

Step 1: Make a table of values.

x									
y									

Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points on each side of $x = 1$.



Domain: All real #s except for 1.

Range: All real #s except for 0.

On Your Own

Find the excluded value of the function.

1. $y = \frac{3}{2x}$

2. $y = \frac{1}{x - 4}$

3. $y = \frac{8}{3x + 1}$

Graph the function. Describe the domain and range.

4. $y = -\frac{8}{x}$

5. $y = \frac{1}{x + 2}$

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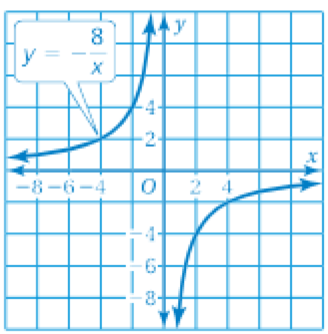
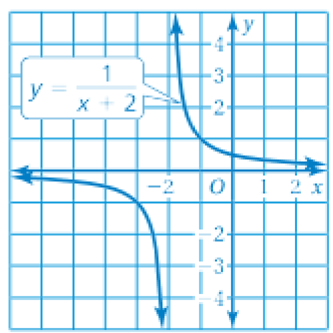


The domain is all real numbers except 0 and the range is all real numbers except 0.

5. $y = \frac{1}{x + 2}$



The domain is all real numbers except −2 and the range is all real numbers except 0.

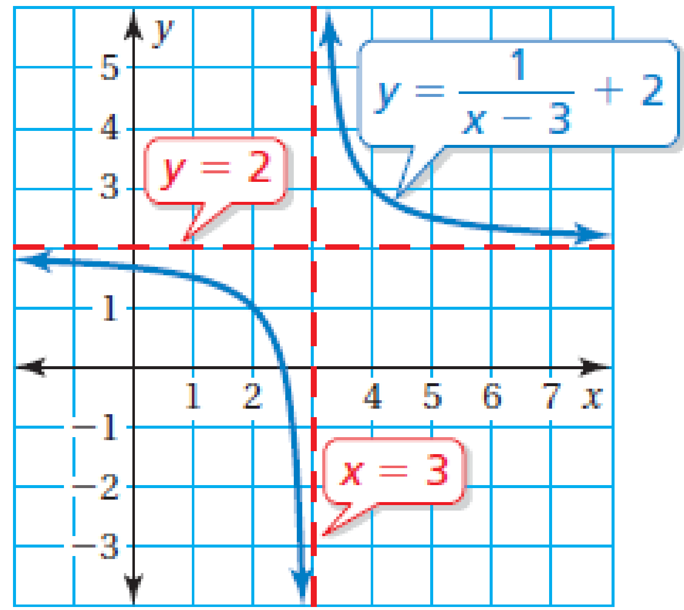
Asymptotes

- Places on the graph the function will approach, but will never touch.

Key Idea

Asymptotes

The graph of a rational function of the form $y = \dfrac{a}{x - h} + k$, where $a \neq 0$, has a vertical asymptote $x = h$ and a horizontal asymptote $y = k$.



EXAMPLE 3 Identifying Asymptotes

Identify the asymptotes of the graph of $y = \dfrac{1}{x-2} - 4$. Then describe the domain and range.

$$y = \frac{1}{x-2} - 4$$

Horizontal Asymptote: $y = -4$

Vertical Asymptote: $x = 2$

Domain: All real #s except for 2.

Range: All real #s except for -4.

On Your Own

Identify the asymptotes of the graph of the function. Then describe the domain and range.

7. $y = \frac{2}{x} + 1$

$x = 0, y = 1;$

The domain is all real numbers except 0 and the range is all real numbers except 1.

8. $y = \frac{1}{x + 5}$

$x = -5, y = 0;$

The domain is all real numbers except -5 and the range is all real numbers except 0.

EXAMPLE 4 Comparing Graphs of Rational Functions

Graph $y = \frac{1}{x+2} + 3$. Compare the graph to the graph of $y = \frac{1}{x}$.

Pay attention to the transformation clues!

$$y = \frac{a}{x - h} + k$$

vertical translation

(-k = down, +k = up)

horizontal translation

(+h = left, -h = right)

The graph of $y = \frac{1}{x+2} + 3$ is a translation 3 units up and 2 units left of the graph of $y = \frac{1}{x}$.

EXAMPLE 5 Real-Life Application

The French club is planning a trip to Quebec City. The function $y = \frac{800}{x+2} + 400$ represents the cost y (in dollars) per student when x students and 2 chaperones go on the trip. How many students must go on the trip for the cost per student to be about $450?

Step 1: Substitute 450 for y.

$$450 = \frac{800}{x + 2} + 400$$

Step 2: Solve for x.

$$50 = \frac{800}{x + 2}$$

$$50x + 100 = 800$$

$$50x = 700$$

$$x = 14$$

About 14 students must go on the trip for the cost per student to be about $450.