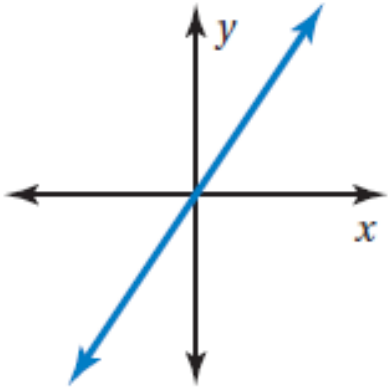
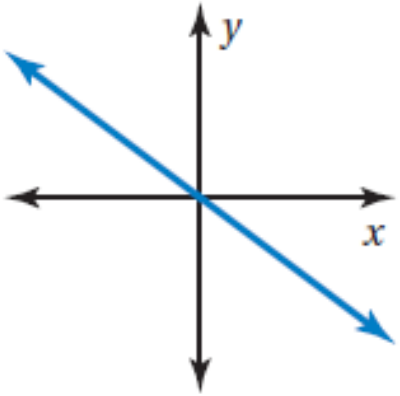
Direct and Inverse Variation
11.1

Direct Variation

- When we talk about a direct variation, we are talking about a relationship where as x increases, y increases or decreases at a CONSTANT RATE.
- Two quantities x and y show direct variation when $y = kx$, where k is a nonzero constant.
- The ratio $\frac{y}{x}$ is constant.
- All direct variation graphs go through the origin.



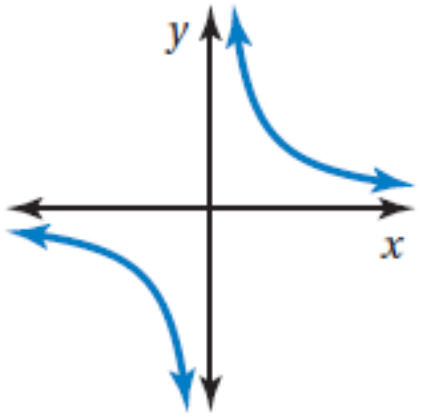
$y = kx,\ k > 0$



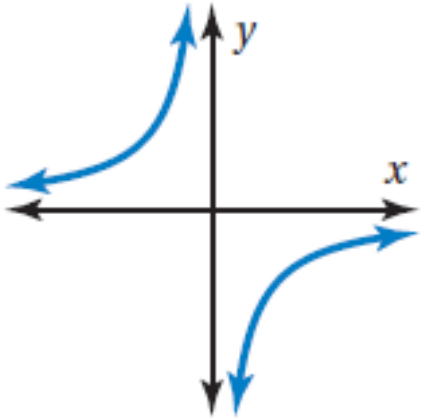
$y = kx,\ k < 0$

Inverse Variation

- In an inverse relationship as one value goes up, the other goes down.
- Two quantities x and y show inverse variation when $y = \frac{k}{x}$, where k is a nonzero constant.
- The product xy is constant.



$y = \frac{k}{x}, k > 0$



$y = \frac{k}{x}, k < 0$

EXAMPLE 1 Identifying Direct and Inverse Variation

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.

a.

x	1	2	3	4
y	5	10	15	20

Direct variation. The ratio $\frac{y}{x}$ is constant.

b. $4xy = -4$

Inverse variation. The equation can be written in the form of $y = \frac{-1}{x}$.

EXAMPLE 2 Writing and Graphing a Direct Variation Equation

1. Plug in the known values for x and y into the model: y = kx.
2. Solve for k.
3. Now write the model y = kx and replace k with the number.

Example: **The variable y varies directly with x. When $x = 12$, $y = -6$. Write and graph a direct variation equation that relates x and y.**

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$y = kx$ Write the direct variation equation.

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$-6 = k(12)$ Substitute 12 for x and -6 for y.

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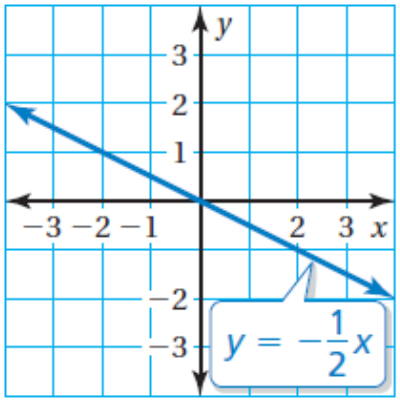
$y = kx$ Write the direct variation equation.

$-6 = k(12)$ Substitute 12 for x and -6 for y.

$-\frac{1}{2} = k$ Divide each side by 12.

So, the equation that relates x and y is $y = -\frac{1}{2}x$.



EXAMPLE 3 Writing and Graphing an Inverse Variation Equation

1. Plug in the known values for x and y into the model: $y = \frac{k}{x}$.
2. Solve for k.
3. Now write the model $y = \frac{k}{x}$ and replace k with the number.

Example: **The variable y varies inversely with x. When $x = 2, y = 5$.**

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Example: **The variable y varies inversely with x. When $x = 2, y = 5$.**

$y = \frac{k}{x}$ Write the inverse variation equation.

$5 = \frac{k}{2}$ Substitute 2 for x and 5 for y.

EXAMPLE 3 Writing and Graphing an Inverse Variation Equation

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2. Solve for k.
3. Now write the model $y = \frac{k}{x}$ and replace k with the number.

Example: **The variable y varies inversely with x. When $x = 2, y = 5$.**

$y = \frac{k}{x}$ Write the inverse variation equation.

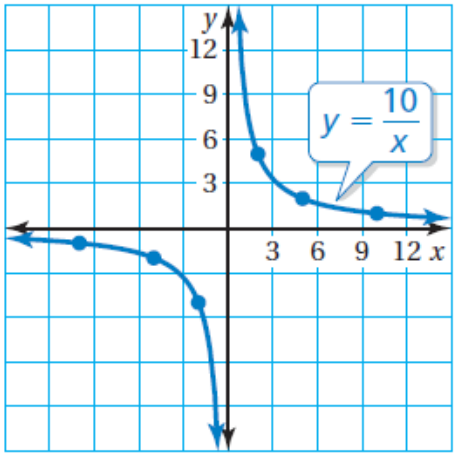
$5 = \frac{k}{2}$ Substitute 2 for x and 5 for y.

$10 = k$ Multiply each side by 2.

So, an equation that relates x and y is $y = \frac{10}{x}$.

Make a table of values and graph the ordered pairs.

x							
y							



EXAMPLE 4 Identifying Inverse Variation

Which situation represents inverse variation?

A

Number of tickets, x	1	2	3
Total cost, y	7.50	15	22.50

No.

B

Number of pounds, x	1	2	3
Total earned, y	0.50	1	1.50

No.

C

Number of people, x	1	2	3
Cost per person, y	600	300	200

Yes.

D

Number of songs, x	1	2	3
Total cost, y	0.99	1.98	2.97

No.

On Your Own

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.

1.

x	1	2	3	4
y	24	12	8	6

Inverse variation: The product xy is constant.

2. $y = 3x + 1$

Neither. The equation cannot be written in the form of $y = kx$ or $y = \frac{k}{x}$.

3. The variable y varies directly with x. When $x = 3$, $y = 15$. Write a direct variation equation that relates x and y.

$$y = 5x.$$

4. The variable y varies inversely with x. When $x = 5$, $y = 4$. Write an inverse variation equation that relates x and y.

$$y = \frac{20}{x}.$$