

Using
The Pythagorean
Theorem
10.4

The converse of a statement switches the hypothesis and the conclusion.

Statement:

If $p = q$.

Converse of the Statement:

If $q = p$.

The converse of the Pythagorean Theorem helps you to find out if a triangle is right.

Basically, the converse states that whenever the sum of the squares of two sides equal to the square of the third side of the triangle, the triangle is a right triangle.

For example, given the following 3 sides, is the triangle right?

4, 5, 3

Is $5^2 = 4^2 + 3^2$?

$5^2 = 25$ and $4^2 + 3^2 = 16 + 9 = 25$

Since $25 = 25$, the triangle is a right triangle.

Other examples:

1) Do the sides 6, 8, and 10 form a right triangle? -

2) Do the sides 9, 12, and 15 form a right triangle?

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$$\text{Is } 10^2 = 6^2 + 8^2 ?$$

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$$10^2 = 100 \text{ and } 6^2 + 8^2 = 36 + 64 = 100$$

2) Do the sides 9, 12, and 15 form a right triangle?

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2) Do the sides 9, 12, and 15 form a right triangle?

$$\text{Is } 15^2 = 9^2 + 12^2 ?$$

$$15^2 = 225 \text{ and } 12^2 + 9^2 = 144 + 81 = 225$$

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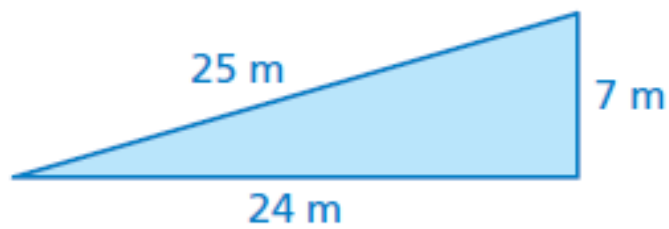
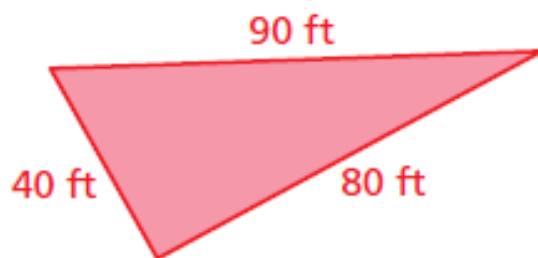
$$15^2 = 225 \text{ and } 12^2 + 9^2 = 144 + 81 = 225$$

Since $225 = 225$, the triangle is a right triangle.

Each of these examples is what is called a Pythagorean Triple, which is a set of 3 positive integers a , b , and c , where $a^2 + b^2 = c^2$.

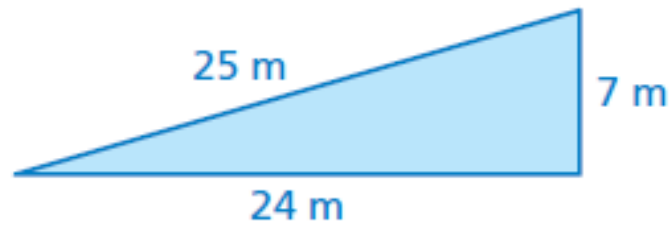
EXAMPLE**1****Identifying a Right Triangle**

Tell whether each triangle is a right triangle.

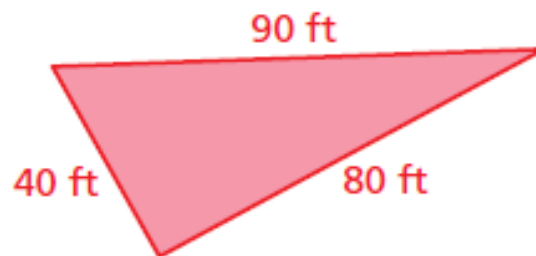
a.**b.**

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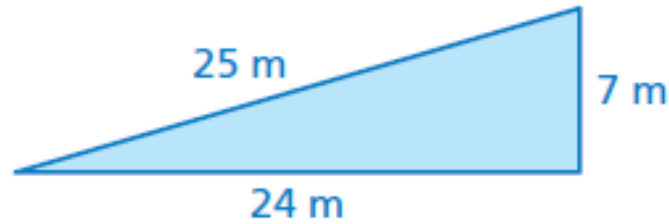
a.

$$a^2 + b^2 = c^2$$
$$7^2 + 24^2 \stackrel{?}{=} 25^2$$

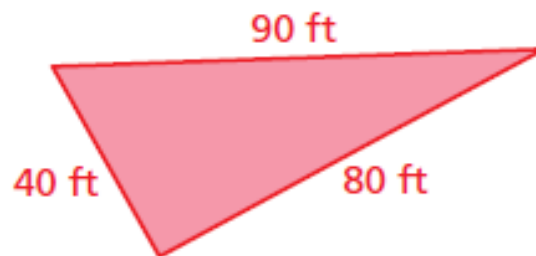
b.

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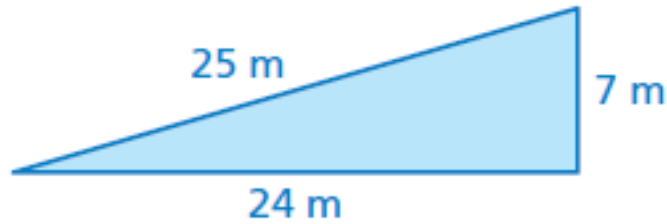
a.

$$a^2 + b^2 = c^2$$
$$7^2 + 24^2 \stackrel{?}{=} 25^2$$
$$49 + 576 \stackrel{?}{=} 625$$

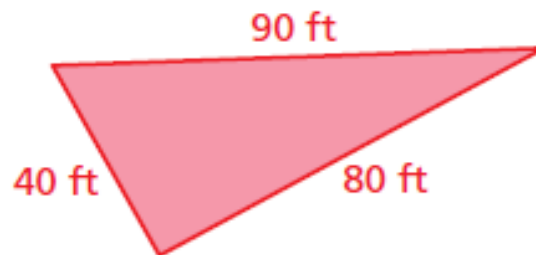
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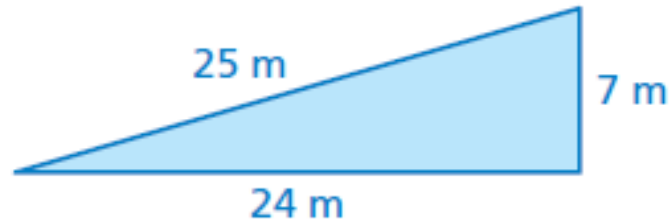
a.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 7^2 + 24^2 &\stackrel{?}{=} 25^2 \\ 49 + 576 &\stackrel{?}{=} 625 \\ 625 &= 625 \quad \checkmark \end{aligned}$$

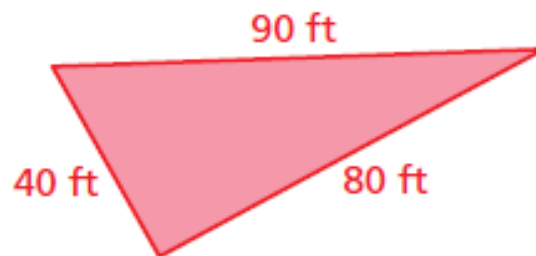
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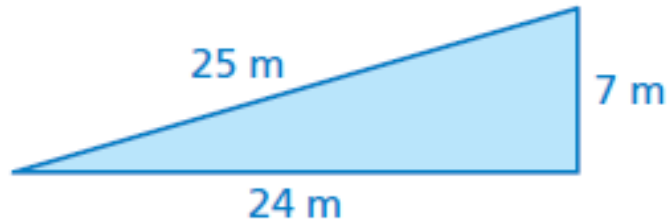
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b.

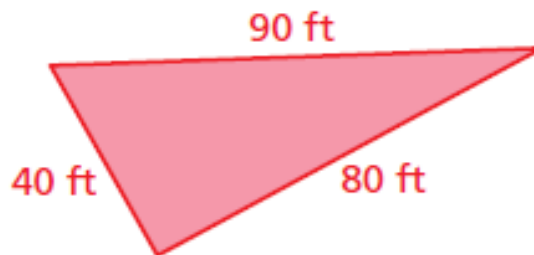
$$a^2 + b^2 = c^2$$

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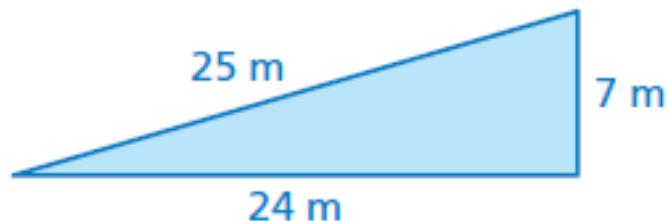
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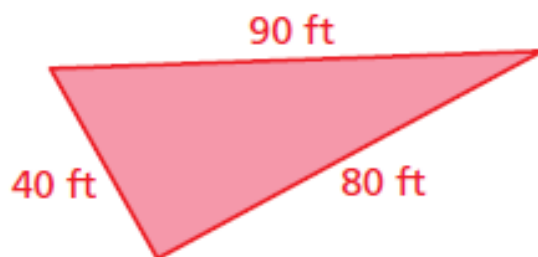
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 40^2 + 80^2 &\stackrel{?}{=} 90^2 \end{aligned}$$

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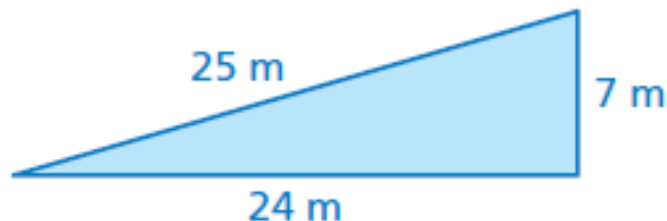
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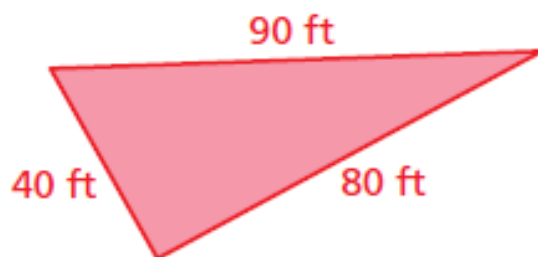
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 40^2 + 80^2 &\stackrel{?}{=} 90^2 \\ 1600 + 6400 &\stackrel{?}{=} 8100 \end{aligned}$$

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$$\begin{aligned}a^2 + b^2 &= c^2 \\7^2 + 24^2 &\stackrel{?}{=} 25^2 \\49 + 576 &\stackrel{?}{=} 625 \\625 &= 625 \quad \checkmark\end{aligned}$$

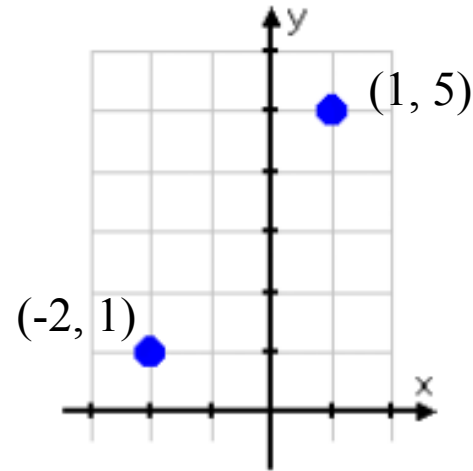
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$$\begin{aligned}a^2 + b^2 &= c^2 \\40^2 + 80^2 &\stackrel{?}{=} 90^2 \\1600 + 6400 &\stackrel{?}{=} 8100 \\8000 &\neq 8100 \quad \times\end{aligned}$$

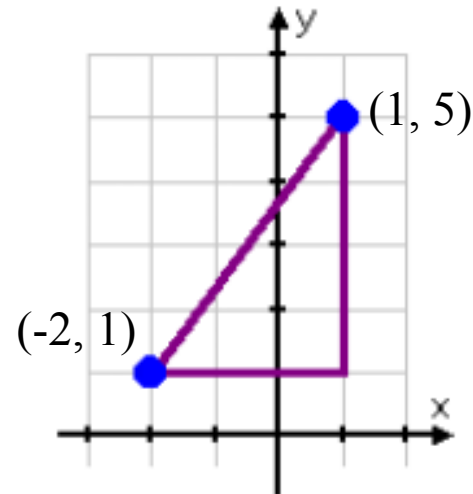
The Distance Formula

The Distance Formula is a variation of the Pythagorean Theorem. Here's how we get from the one to the other.

Suppose you're given these two points $(-2, 1)$ and $(1, 5)$, and they want you to find out how far apart they are. The points look like this:



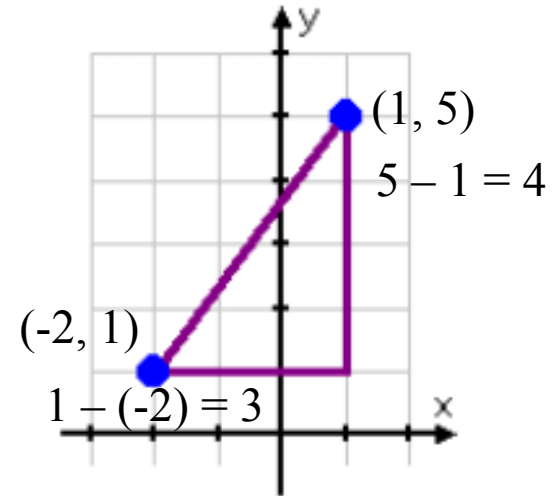
You can draw in the lines that form a right-angled triangle using these points as two corners.



The Distance Formula

The Distance Formula is a variation of the Pythagorean Theorem. Here's how we get from the one to the other.

It's easy to find the length of the horizontal and vertical sides of the right triangle: just subtract the x-values and the y-values.



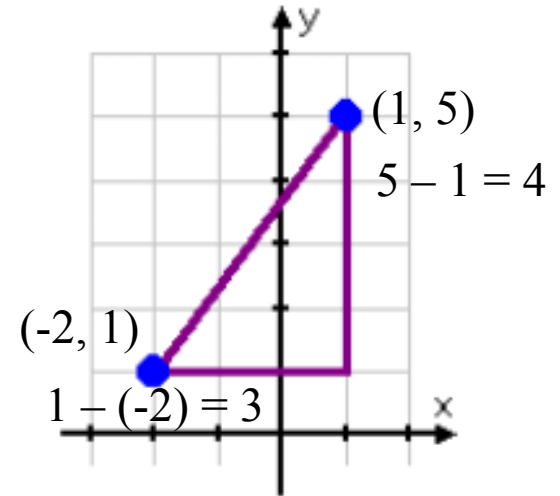
Then use the Pythagorean Theorem to find the length of the 3rd side (which is the hypotenuse of the right triangle).

$$c^2 = a^2 + b^2$$

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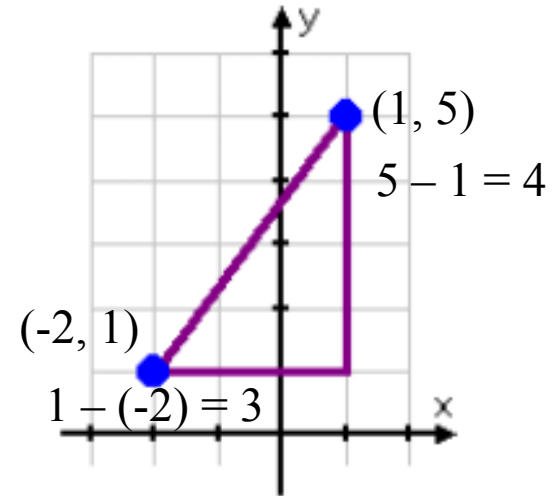
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$$c^2 = a^2 + b^2 \quad \dots \text{so:} \quad c^2 = (5 - 1)^2 + (1 - (-2))^2$$

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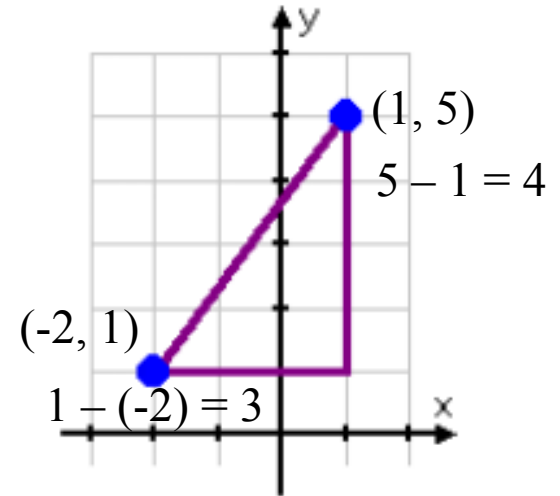
$$c^2 = a^2 + b^2 \quad \dots \text{so:} \quad c^2 = (5 - 1)^2 + (1 - (-2))^2$$

$$c = \sqrt{(5 - 1)^2 + (1 - (-2))^2}$$

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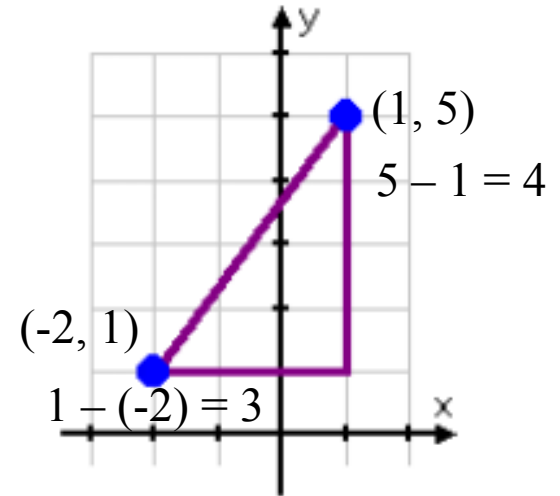
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$$\begin{aligned} c &= \sqrt{(5 - 1)^2 + (1 - (-2))^2} \\ &= \sqrt{(4)^2 + (3)^2} \end{aligned}$$

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$$= \sqrt{(4)^2 + (3)^2}$$

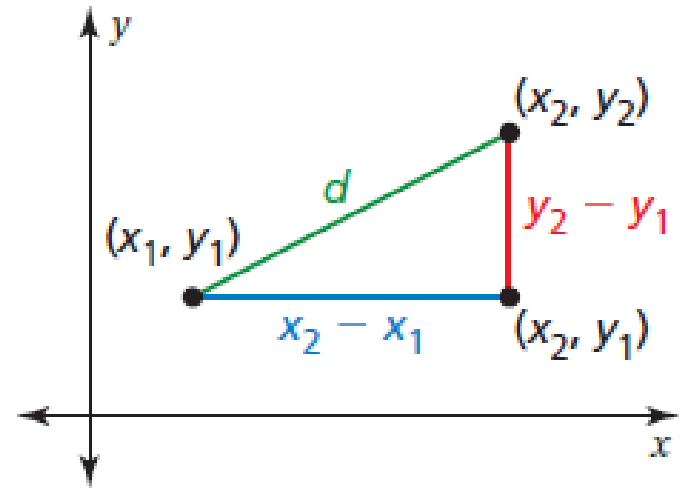
$$= \sqrt{16 + 9} = \sqrt{25} = 5$$

Key Idea

Distance Formula

The distance d between any two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



EXAMPLE 2 Finding the Distance Between Two Points

Find the distance between $(-3, 5)$ and $(2, -1)$.

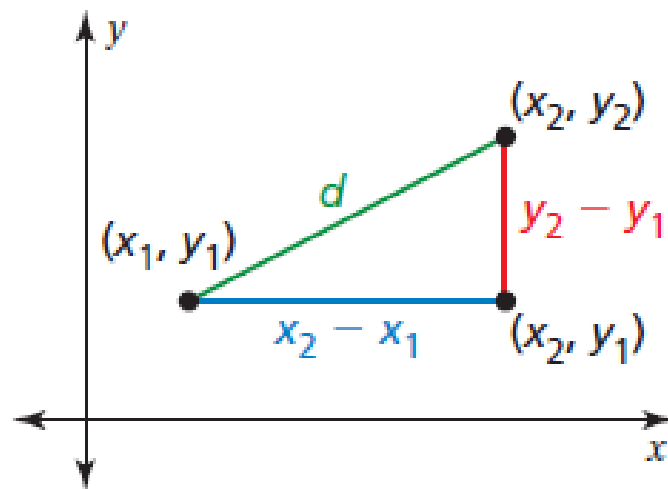
Let $(x_1, y_1) = (-3, 5)$ and $(x_2, y_2) = (2, -1)$.

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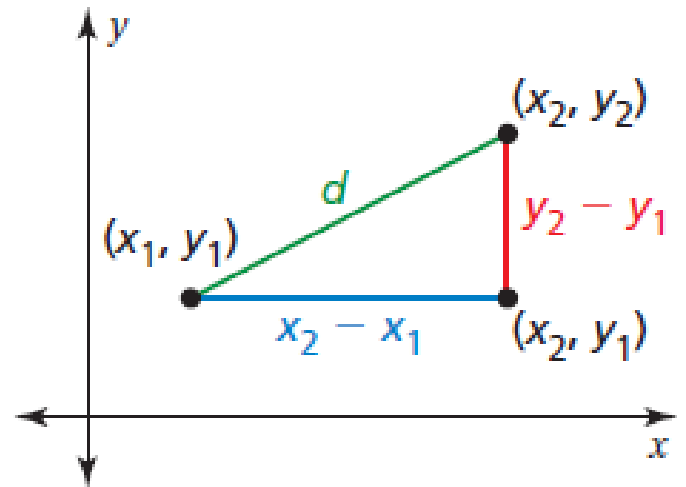
Write the distance formula:

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Write the distance formula:

$$= \sqrt{[2 - (-3)]^2 + (-1 - 5)^2}$$

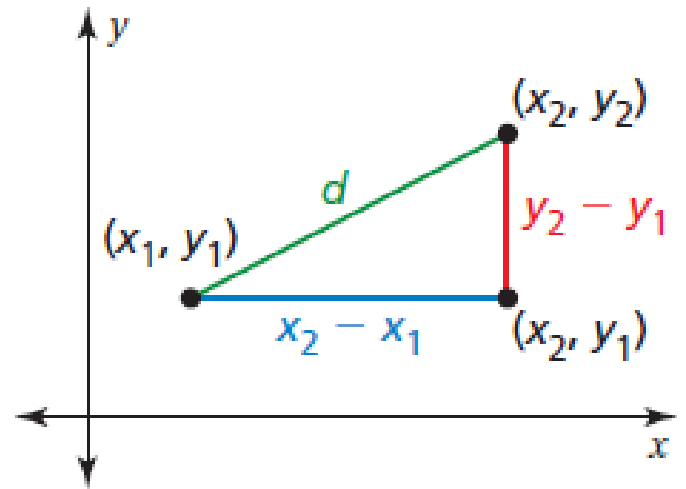
Substitute.

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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[2 - (-3)]^2 + (-1 - 5)^2}$$

$$= \sqrt{5^2 + (-6)^2}$$

Write the distance formula.

Substitute.

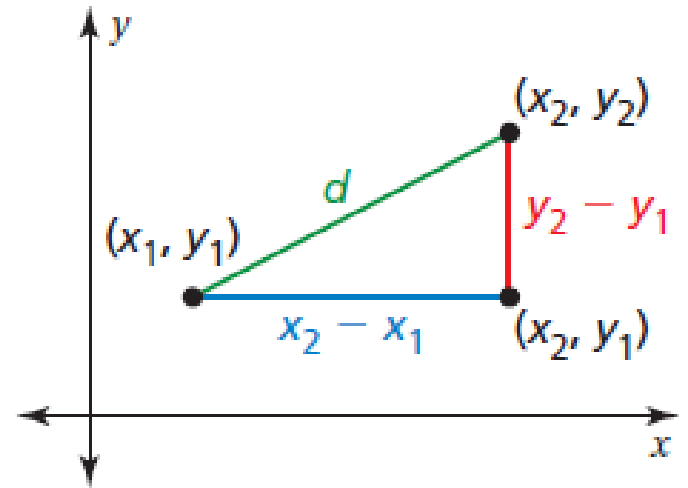
Simplify.

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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[2 - (-3)]^2 + (-1 - 5)^2}$$

$$= \sqrt{5^2 + (-6)^2}$$

$$= \sqrt{25 + 36}$$

Write the distance formula.

Substitute.

Simplify.

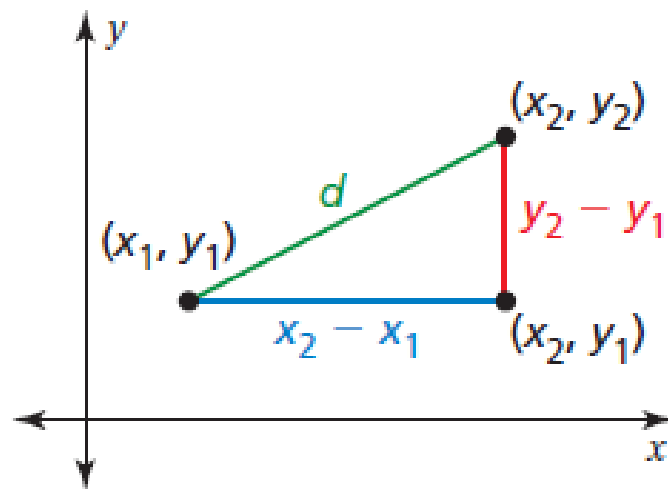
Evaluate powers.

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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[2 - (-3)]^2 + (-1 - 5)^2}$$

$$= \sqrt{5^2 + (-6)^2}$$

$$= \sqrt{25 + 36}$$

$$= \sqrt{61}$$

Write the distance formula.

Substitute.

Simplify.

Evaluate powers.

Add.