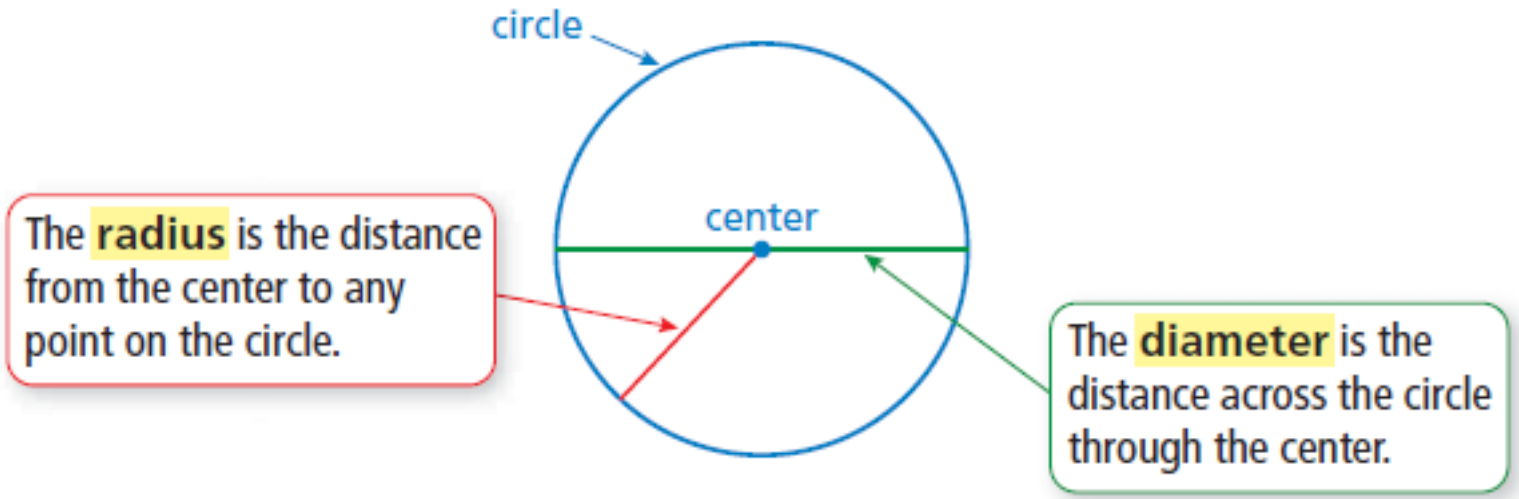
8.1 Circles and Circumference

A **circle** is the set of all points in a plane that are the same distance from a point called the **center**.



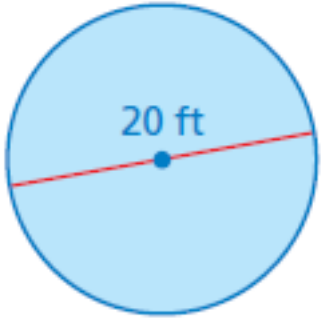
Radius and Diameter

Words The diameter d of a circle is twice the radius r. The radius r of a circle is one-half the diameter d.

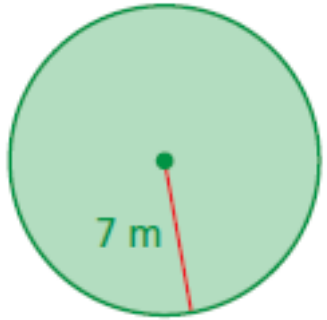
Algebra **Diameter:** $d = 2r$ **Radius:** $r = \frac{d}{2}$

EXAMPLE 1 Finding a Radius and a Diameter

a. The diameter of a circle is 20 feet. Find the radius.



b. The radius of a circle is 7 meters. Find the diameter.



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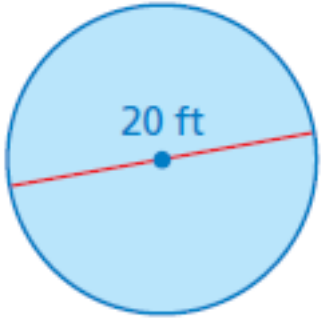

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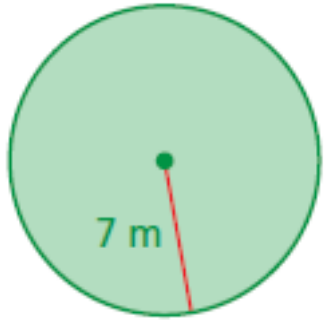

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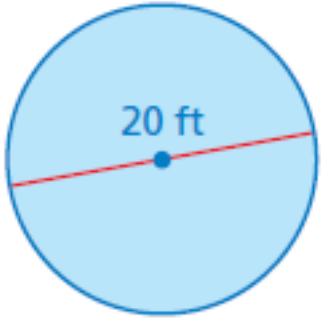

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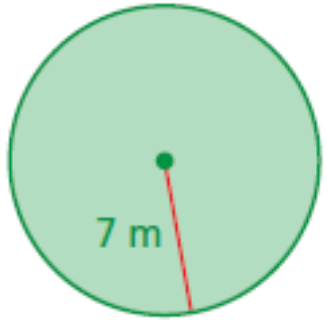

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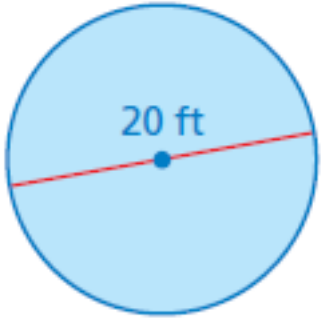

$$r = \frac{d}{2} \qquad \text{Radius of a circle}$$

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$$= 10 \qquad \text{Divide.}$$

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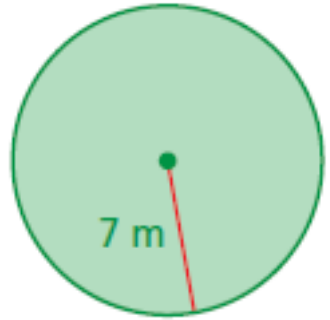

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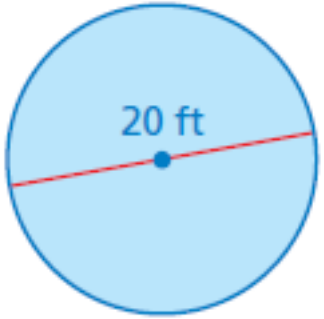

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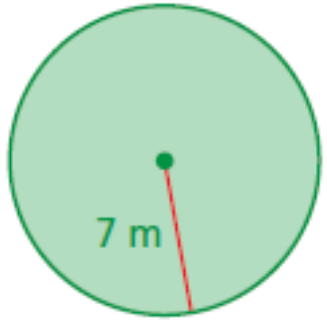

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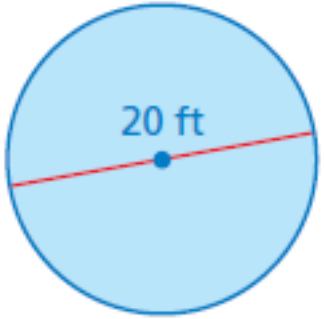

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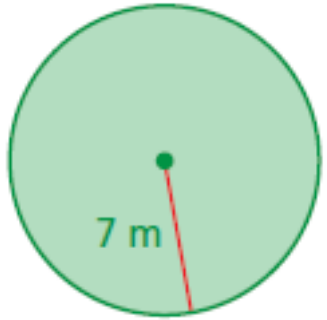

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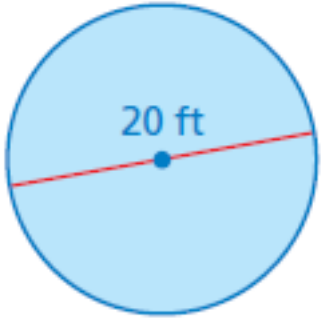

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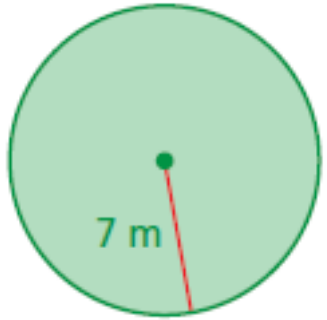

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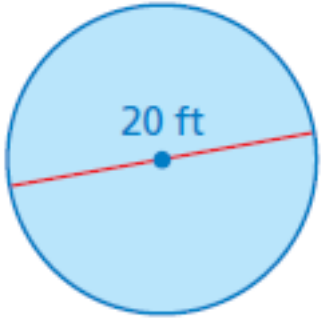

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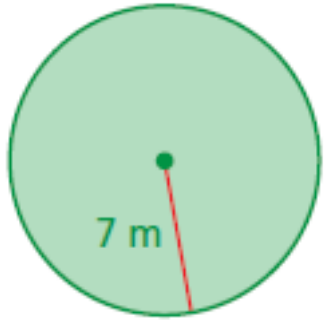

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$= 14$ Multiply.

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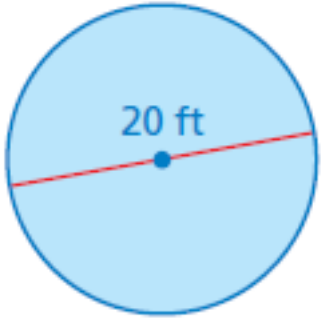

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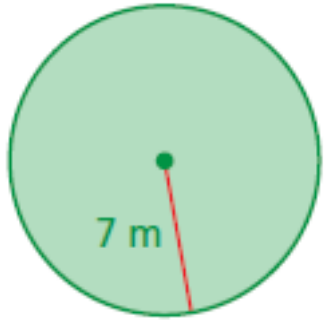

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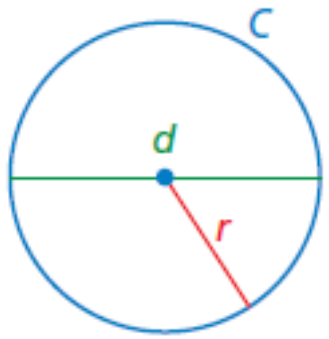
$= 14$ Multiply.

∴ The diameter is 14 meters.

The distance around a circle is called the **circumference**. The ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for *every* circle and is represented by the Greek letter π, called **pi**. The value of π can be approximated as 3.14 or $\frac{22}{7}$.



Circumference of a Circle

Words The circumference C of a circle is equal to π times the diameter d or π times twice the radius r.

Algebra $C = \pi d$ or $C = 2\pi r$

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a. Find the circumference of the flying disc. Use 3.14 for π.

b. Find the circumference of the watch face. Use $\frac{22}{7}$ for π.

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a. Find the circumference of the flying disc. Use 3.14 for π.

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b. Find the circumference of the watch face. Use $\frac{22}{7}$ for π.

$C = \pi d$ Write formula for circumference.

$\approx \frac{22}{7} \cdot 28$ Substitute $\frac{22}{7}$ for π and 28 for d.

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$\approx \frac{22}{7} \cdot 28$ Substitute $\frac{22}{7}$ for π and 28 for d.

$= 88$ Multiply.

$\therefore$ The circumference is about 88 millimeters.

EXAMPLE 3 Estimating a Diameter

$C = 31.4$ in.

The circumference of the roll of caution tape decreases 10.5 inches after a construction worker uses some of the tape. Which is the best estimate of the diameter of the roll after the decrease?

(A) 5 inches (B) 7 inches (C) 10 inches (D) 12 inches

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$C = \pi d$	Write formula for circumference.
$20.9 \approx 3.14 \cdot d$	Substitute 20.9 for C and 3.14 for π.
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$7 = d$	Divide each side by 3.

$\therefore$ The correct answer is (B).

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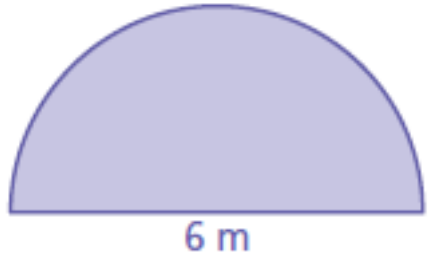
A semicircle is one-half of a circle. Find the perimeter of the semicircular region.



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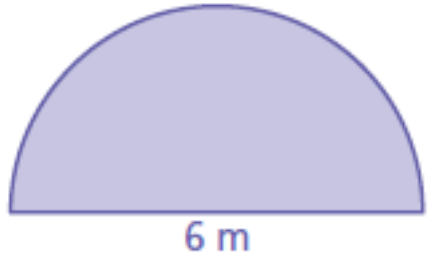

The straight side is 6 meters long. The distance around the curved part is one-half the circumference of a circle with a diameter of 6 meters

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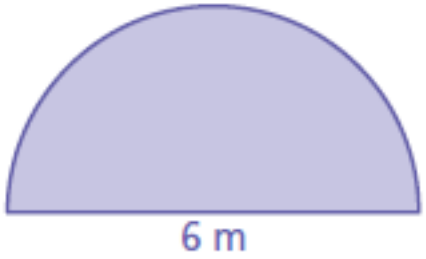

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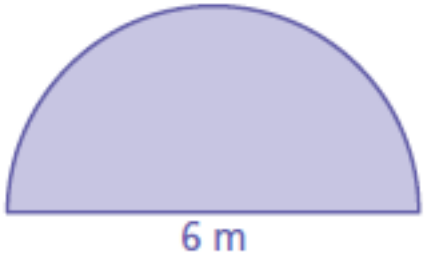

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$$\approx \frac{3.14 \cdot 6}{2}$$ Substitute 3.14 for π and 6 for d.

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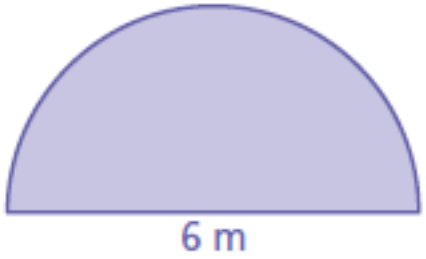

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$$= 9.42$$ Simplify.

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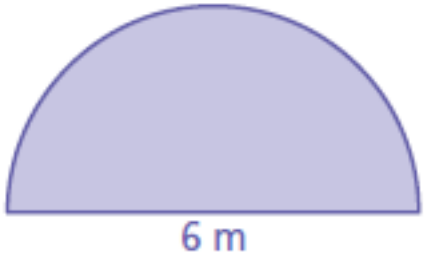

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$$= 9.42$$ Simplify.

$$= 9.42 + 6$$ Add the distance across the diameter.

$\therefore$ So, the perimeter is about $6 + 9.42 = 15.42$ meters.